Asynchronous, Distributed Optimisation for Cooperative Agents in a Smart Grid

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To Ann, for her patience and unfailing support.

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Declarations

Declaration of Originality

This thesis contains no material which has been accepted for a degree or diploma by the University or any other institution, except by way of background information and duly acknowledged in the thesis, and to the best of my knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright.

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Statement of Co-authorship

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Paper 3, Smart Grid Optimization Through Asynchronous, Distributed Primal Dual Iterations:
Located in chapter 4
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Paper 4, Asynchronous Consensus for Optimal Power Flow Control in Smart Grid with Zero Power Mismatch:
Located in chapter 5
Candidate was the primary author and with author 1 contributed to the idea, its formalisation and development. Candidate contributed approximately 80% and author 1 20%.

Paper 5, An Asynchronous, Distributed Protocol for DC Power Management in a Smart Building:
Located in chapter 6
Candidate was the primary author and with author 1 contributed to the idea, its formalisation and development. Candidate contributed approximately 80% and author 1 20%.

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Abstract

This thesis addresses a number of issues present in the modern and future power distribution system where high penetration of distributed generation (DG) and smart sensors change the environment in which power flow must be managed. The shift in balance of power supply from the centralized to the distributed can lead to network constraint breaches such as voltage and frequency limits, fault ride through capability, system security, reliability and stability. Common regulation approaches may be inadequate in addressing these network regulation problems, leading to inefficient use of DG and unnecessary high voltage (HV) grid imports. Furthermore, the increase in intelligent Smart Grid components will lead to the transmission and processing of large volumes of data making the optimal control of a network a more challenging problem. Optimisation methods must take into consideration the increase and distributed nature of data, and account for data synchronization, latency, and privacy issues.

However, if these challenges can be overcome, then the increase in the controllability and observability of smart grid components, such as distributed generators, storage and controllable demands, offers great potential for the improvement of network optimality and stability. In this thesis, a set of innovative distributed algorithms are presented that solve the optimal power flow problem of a distribution network featuring advanced nodal monitoring and control of DG, storage and loads. These algorithms exploit the network structure to produce iterative solutions to solve the global optimisation problem. They are carefully developed taking into account the realistic limitations where each node can only exchange information with adjacent neighbours but does not have sufficient information about the other nodes in a large scale system.

Two partitioning strategies are considered which aim to improve the structure of the communication and control subsystem in order to better facilitate optimal control. The first strategy measures subnet optimality according to the minimisation of mismatch between DG power and local demand, therefore maximising DG utilisation, minimising line loss, and min-

imising HV grid imports. The second strategy is based on sets of strongly coupled buses, where the coupling of buses is characterised according to the potential for a change in power at one bus to impact the state estimation error at another, therefore improving solution optimality.

Subsequently, a distributed predictive optimal control algorithm is proposed, through the method of approximate dynamic programming, that utilises a central coordinator to improve network state estimation and control sequence optimality. The centrally coordinated solution has the benefit of a near optimal solution without burdening controllers with the high-dimensional state of the entire distribution network, but rather utilising only a summary of global information.

Improvements to the centrally coordinated scheme are then developed through a fully distributed optimal power flow algorithm that requires no central coordination. The fully distributed approach maintains the reduced computational requirements of controllers but improves on the centrally coordinated configuration by restricting data communication to local neighbourhoods. Three variants of the distributed OPF solution are suggested for application to three distinct scenarios: Optimal DG control in a distribution network, DG optimal control in an islanded distribution network, and optimal power management in a home energy management system. These three approaches address significant issues associated with distributed control. An optimal solution to the global problem is achievable in each case, and iterations of the algorithm are shown to be stable and convergent through the use of an augmented Lagrange formulation. Global information necessary for a feasible solution is shared through the development of a new asynchronous consensus protocol, and a communication protocol is presented to enable instantiation, execution and conclusion of fully distributed optimisation sessions.

For each studied approach, algorithms are carefully developed that concisely define the method of application. For each presented algorithm, a reasonable amount of computer simulation is applied to verify their applicability to a range of relevant scenarios. The simulations study the algorithms' convergence and scalability, solutions' optimality in a local sense, and solution feasibility. In each case the simulations successfully demonstrate the presented methods' practicality for the studied scenarios.

Contents

List of Tables

List of Figures

CHAPTER 1

Introduction

Intelligent power networks provide opportunities for improved power supply through solutions that draw from increased communication and information processing capabilities present in the smart grid. Additionally, the increased monitoring and control within components of the smart grid may be employed to improve observability, controllability and optimality of distributed components such as distributed generators (DG), storage, sensors and smart home devices. The shift towards these distributed components gives opportunities for improved cooperation between smart grid agents, however it also requires increased monitoring and control to avoid network constraint breaches, in particular in the case of high DG penetration. The integration of communication and intelligence into a distribution network also gives rise to new problems relating to network scalability and high data volumes, and new solutions in the form of distributed monitoring and control are required. In this section these problems are formally defined in the context of the thesis' research, and the thesis contributions are presented at a high level.

To provide an overview of this thesis, the associated problems addressed are defined in Section 1.1, its research outcomes are presented in Section 1.2, its associated publications in Section 1.3, and its outline in Section 1.4.

1.1 Thesis Problems

Recent research into the advancements and future potential in power networks has seen a shift from centralised scheduling and control methods to various distributed approaches (for numerous examples, refer to Section 2.2). These distributed approaches aim to address the issues associated with high data volumes, and to reduce centralized communication and processing burdens, while maintaining privacy and optimal or near-optimal grid operation.

As such, this thesis' focus is to investigate and provide solutions to the following problems with a focus on optimal power flow in smart distribution networks:

Problem 1 The logical structure of a traditional distribution network is unlikely to be conducive to the convenient application of distributed optimisation methods.

Problem 2 Distributed optimisation approaches do not accurately consider the power flow and regulatory constraints within smart distribution networks.

The objective of this thesis, with regards to Problem 1, is to identify the relevant aspects of network structure, and to find optimal network partitions, for the purpose of optimal network operation. Additionally, the sub-problem of shifting optimal partition structure due to changes in network state over time will also be addressed. Problem 1 is complex to solve due to the dynamic nature of an active distribution network and the optimality of a partitioned structure will vary depending on the operating state.

The objective of this thesis, with regards to Problem 2, is to develop distributed optimal power flow algorithms that both select a feasible and an optimal or near optimal operating state. Additionally, the sub-problems of distributed handling of global values of interest, and asynchronous communication without central coordination will be addressed. Problem 2 is difficult to solve without knowledge of the full network state since solution feasibility at any bus is dependent on the state of all other network buses. The difficulty of the problem is further increased due to the non-convexity of the optimal power flow problem.

1.2 Research Outcomes

The research focus of this thesis is to develop improved distributed algorithms for solutions to important optimisation problems present in the future smart grid. Through the formulation of multi-agent systems, network structure is exploited to solve the economic dispatch of distributed generators, storage and loads. This provides the benefit of more extensible and scalable smart grids that are not overburdened by high data volumes.

In order to develop distributed solutions capable of efficient optimisation, the distributed agents must first be defined. Two approaches are taken to address this requirement: Network partitioning to form zones, and application of a non-hierarchical multi-agent system (MAS). The presented partitioning algorithms extend the method of sensitivity matrix decomposition to account for the forecast balance of generation capacity and loads within each zone, and voltage estimation errors due to power flow approximations. This partitioning approach better enables zone controllers to optimally manage resources and to better approximate state within their scope of control and observation.

Application of a non-hierarchical MAS to the optimal power flow (OPF) problem is explored in depth through three formulations, each of which is specifically applicable to different scenarios and addresses different challenges specific to each case. Careful formulation of the economic dispatch problem within the smart grid context allows for a fully distribution OPF solution to be developed. The presented solution is capable of calculating power flow without any central coordination, and of reaching an optimal solution of the global OPF problem, accounting for voltage, current and power limits, through neighbourhood communication only.

To assist the solution of OPF within an isolated network, a new asynchronous averaging consensus algorithm is developed. The consensus algorithm enables agents within the network to discover the mismatch between generated power and loads in cooperation with the asynchronous distributed OPF solution. The developed consensus algorithm features some benefits over its predecessors: No leader agent is required, neighbourhood synchronisation is not required, neighbouring agents are implicitly paired asynchronously, and an average consensus is achieved and tracked.

Finally, an application level communication protocol is developed that enables the MAS to initiate, conduct and conclude optimisation sessions without the need for a central coordinator or synchronisation. The protocol is an enabler for implementing the presented distributed optimisation algorithms in a practical scenario. A home energy management system (HEMS) is chosen for a demonstration of the protocol applied to managing the distributed optimisation.

The following points highlight the contributions of this work:

1. Partitioning systems are proposed that improve upon previous work by accounting for forecasts and power balance, and minimising likely

voltage errors.

2. A centrally coordinated, distributed smart grid optimisation algorithm is presented that accounts for uncertain future states, the capacity of agents to manage information within their zones, the likely voltage estimation errors, and the scalability of the solution.

3. An asynchronous, distributed algorithm built on a primal dual iterative optimisation process is presented that is able to solve the global OPF problem of optimal DG control without any central coordination or synchronisation.

4. An asynchronous averaging consensus algorithm is developed which requires no leader agent, no synchronisation, and no explicit agent pairing, and is then combined with the asynchronous, distributed OPF solution.

5. An asynchronous inter-agent communication protocol is developed that enables agents to begin, execute and conclude optimisation sessions.

1.3 Publications

The research publications listed below were written during my PhD candidature and submitted to or published in fully refereed international conference proceedings or journals. Chapters 3 to 7 are based on the content of these papers, which were the contribution of the author, and are organised in a manner consistent with the thesis context.

- B. Millar, D. Jiang, and M. E. Haque. "A novel partitioning strategy for distribution networks featuring many small scale generators". *2013 IEEE PES Innovative Smart Grid Technologies Conference (ISGT)*. Feb. 2013, pp. 1–6. DOI: 10.1109/ISGT.2013.6497813

- B. Millar, D. Jiang, and M. E. Haque. "Constrained coordinated distributed control of smart grid with asynchronous information exchange". *Journal of Modern Power Systems and Clean Energy* 3.4 (Dec. 2015), pp. 512–525. ISSN: 2196-5420. DOI: 10.1007/s40565-015-0168-1. URL: https://doi.org/10.1007/s40565-015-0168-1

- B. Millar and D. Jiang. "Smart Grid Optimization Through Asynchronous, Distributed Primal Dual Iterations". *IEEE Transactions on Smart Grid* 8.5 (Sept. 2017), pp. 2324–2331. ISSN: 1949-3053. DOI: 10.1109/TSG.2016.2522970

- B. Millar and D. Jiang. “Asynchronous Consensus for Optimal Power Flow Control in Smart Grid with Zero Power Mismatch”. *Accepted for Publication in Journal of Modern Power Systems and Clean Energy* (2017)

- B. Millar and D. Jiang. “An Asynchronous, Distributed Protocol for DC Power Management in a Smart Building”. *IEEE International Conference on Communication Technology.* Oct. 2017

1.4 Thesis Outline

The advent of the smart grid brings many benefits but also many new challenges. Of interest in this thesis is the challenge presented by large data volumes requiring transmission and processing. The remainder of this thesis investigates distributed solutions to smart grid optimal power flow and related problems, with a particular focus on the thesis problems and objectives defined in Section 1.1.

Chapter 2 presents a review of literature relating to the smart grid, with a particular focus on the thesis problems.

In Chapter 3 a partitioning strategy is introduced in order to enable a sub-layer of control over distributed generators (DG), and contributes to the objective of finding optimal network structure to address Problem 1. The proposed partitioning better enables zone controllers to manage DG, especially in the case of an imbalance between generation and load.

In Chapter 4 a centrally coordinated distributed approximate dynamic program (ADP) with asynchronous information exchange between local and central nodes is presented. The presented methods include algorithms for partitioning and optimal power flow with a focus on the impacts of state changes over time to address Problem 1 and its sub-problem, and Problem 2. The proposed distributed ADP algorithm is able to reduce the optimal power flow (OPF) problem dimensionality, improve local state estimation, and handle delays in information exchange.

In Chapter 5 an asynchronous, localised primal dual method to solving the OPF problem is developed, in order to address Problem 2 and its objective of optimal control. The localised OPF form uses only local and neighbourhood communication allowing for a completely distributed implementation that is able to reach an optimal solution to the global problem without use of a central agent.

In Chapter 6 a new asynchronous averaging consensus protocol is applied to the distributed OPF solution presented in the previous chapter in order to extend the capability of the asynchronous, distributed OPF method. The consensus protocol addresses Problem 2's sub-problem of distributed handling of global values of interest. This allows the inclusion of inseparable global constraints, such as network power mismatch, into the OPF problem.

In Chapter 7 the Home Energy Management Multi-Agent (HEMMA) protocol is developed to enable the coordination of agents within a smart building for the purpose of solving the distributed DC OPF problem. The HEMMA protocol addresses Problem 2's sub-problem of needing an asynchronous communication mechanism without central coordination. The asynchronous protocol allows the distributed optimisation to be managed while being tolerant to network structural changes.

Finally, Chapter 8 concludes the thesis and discusses potential future expansion of this research.

CHAPTER 2

Literature Review

The smart grid has seen extensive research across a broad range of problems in modern literature. The introduction and expansion of communication and computing resources present within a smart grid, provides new opportunities for improved monitoring and control, and in turn can lead to increased observability, controllability, and optimality. These exciting possibilities have lead to an explosion of research in the field of smart grid advanced monitoring and optimal control.

To give a clearer context to these smart grid issues, the components of a smart grid and their associated problems are reviewed in Section 2.1, and a review of distributed smart grid problems and solutions is provided in Section 2.2.

2.1 The Smart Grid – A Brief Review

The future power transmission and distribution system is envisioned to incorporate many intelligent components connected by communication infrastructure. Smart sensor and actuator agents may utilise the communication infrastructure to report network state back to central nodes for monitoring purposes, transmit sensor data to other agents for cooperative processing, and apply the information locally for decentralised optimisation. Such a power network with integrated communication and intelligence is referred to in the literature as a smart grid [6, 7, 8, 9, 10]. The smart grid plays an essential role in fulfilling future power needs and provides the infrastructure and mechanisms to achieve improved reliability, security, economics, efficiency, environmental impact, safety, and automation [11, 12, 9].

In recent years governments and energy utilities have provided incentives to push power supply away from fossil fuel based generation to the use of renewable energy sources [13]. This has in turn seen a dramatic increase in the use of small scale power generation, for example through rooftop solar and other distributed generators (DG). Beyond the reduced environmental impacts of renewable DG, there are numerous benefits to the use of distributed power generators [12]. DG can provide power directly to its neighbourhood and therefore reduce the need for transmission over large distances from high voltage transmission to low voltage distribution. This has the added benefit of reducing power loss due to transmission and ultimately improves performance [14].

However, there are also a number of issues introduced by the presence of DG that must be carefully considered [15, 16, 17]. High penetration of DG is likely to introduce periods of reverse power flow within the distribution network, in particular in residential areas where high solar irradiance coincides with low demand. Older infrastructure is typically not designed for reverse power flow and may cause network operational constraints to be breached, in particular due to voltages exceeding their upper limits. Traditional transmission and distribution networks were not designed to accommodate localised generation and the resultant non-hierarchical power flow between regions. This is expected to be especially evident in rural areas [12]. The result of this is congestion and potentially the need to curtail power generation within the transmission and distribution systems.

In addition to the accommodation of DG into the transmission and distribution networks smart grids also aim to address a range of issues within existing power systems. Existing infrastructure is ageing and is limited in its ability to meet growing demands for power and efficient delivery, especially in the presence of bidirectional power flow [18]. These ageing assets provide limited observability which makes network state estimation unreliable, especially in the presence of intermittent generation and load [19]. This limits the ability of existing networks to support improved optimisation techniques that take advantage of distributed generation, intelligent appliances within homes and businesses, and active demand management by consumers.

As such, smart grid development aims to accommodate a move from traditional hierarchical power generation and delivery structures, to a less centralised system allowing for bidirectional power flow, greater observability and controllability [20], and improved efficiency.

A typical smart grid infrastructure can be divided into three interconnected systems: The energy system, communication system, and information system [9]. The energy system produces, distributes and consumes power within the grid. The communication system links the components of the energy system allowing for sharing of monitoring information and the possibility of remote, cooperative and optimal control. The information system provides the computing infrastructure that intelligently implements analysis and control of components within the energy system, based on the information received through the communication system. Further details are provided in the following Sections.

2.1.1 Energy System

The energy system consists of the power generation, transmission, distribution and storage and the physical infrastructure required to support it. The smart grid infrastructure's energy system incorporates both traditional centralised generation (e.g. fossil fuel and nuclear plants, large scale solar and wind farms), and distributed generation (e.g. rooftop and small-scale solar), producing two-way power flow in order to meet demand. The energy system includes a range of heterogeneous power sources, storage and loads which must operate in a heavily dependent manner, and in the context of a smart grid should work in order to better serve consumers. The following lists some components of the energy system commonly studied in recent research.

Distributed generators (DG): Benefits of DG include the provision of ancillary services [21, 22], reduction of line loss due to localised supply of power [14], increased use of renewable power generation, reduced costs for consumers, improved reliability and voltage support, improved security, reduced reserve requirements and numerous potential environmental benefits [23]. Methods of DG control include through aggregation to form a virtual power plant [9, 24, 25], constrained economic dispatch [26, 27, 28, 29, 30, 31], and overvoltage regulation [32, 33].

Distributed storage: The use of distributed storage devices is set to increase [34]. Applications of distributed storage include peak shaving [35, 36, 37, 38], frequency control support [39], economic dispatch [40, 30, 28, 41, 42], and compensation for renewable generation fluctuations [35, 43]. A review of distributed storage technologies can be found in [44].

Electric vehicles: Plugin electric vehicles (PEV) present an addition significant load, albeit typically at traditional off-peak times. Solutions to manage the addition of PEV to the power system include the coordination of multiple PEV with DG [45, 46, 47, 48], and the use of vehicle batteries as dispatchable storage referred to as Vehicle to Grid (V2G) [49, 50, 51].

2.1.2 Communication System

The communication system consists of the wired and wireless physical infrastructure used for information transmission within the smart grid and also the protocols involved. The communication systems used must account for the range of components within the hierarchy of the energy system, including plant-wide networks for communication between control centres and generation plant equipment, wide area networks for communication between geographically distant smart grid agents, field area networks for transmission of monitoring and control data between controllers and distributed devices, and premise networks for the communication between energy management components within a home energy management system (HEMS). A wide range of existing communication technologies have been suggested as appropriate for application within the smart grid. The following provides a brief review of some of these technologies.

Wireless Technologies: Numerous wireless communication technologies have been suggested for application to the smart grid including Zigbee, Wi-Fi, Bluetooth, 6LowPAN, Z-wave, Wireless Mesh and Cellular. Reviews of these technologies can be found in [52, 10]. A subset of these are emphasised in he following three technologies in this category.

Wireless Technologies - ZigBee: Zigbee has become one of the preferred technologies for communicating meter readings and between intelligent home appliances. Potential issues with ZigBee are its security limitations [53] and potential interference from overlapping Wi-Fi bands [54].

Wireless Technologies - Wireless mesh: Nodes in a wireless mesh network route traffic via their neighbours without central coordination. Applications of wireless mesh technology in the smart grid includes its use in advanced metering infrastructure (AMI) through installation of radio receivers and transmitters into each meter [55, 56, 57, 58].

Wireless Technologies - Cellular Networks: The existing cellular networks can be applied to the transmission of AMI data. Numerous telecommunication companies have already agreed to this application, and new networks are also being set up for the specific purpose of AMI communication [52]. A review of the application of cellular networks to smart grid neighbourhood area network communications can be found in [59].

Wired Technologies: Power line communication (PLC) and digital subscriber lines (DSL) have the benefit of reusing existing infrastructure and are a preferred choice for AMI communications [60, 52]. Reviews of PLC technology, its modelling, and its application to the smart grid can be found in [61, 62].

Smart Meter Communication: Protocols for use within AMI must take into consideration privacy and the potential for large data volumes. Common solutions to these problems include meter data aggregation and encryption, and authentication protocols which reduce data size and preserves privacy through obfuscation [63, 64, 65, 66]. Reviews of security and privacy concerns can be found in [67, 68], and a review of automated meter reading protocols can be found in [69].

Protocols: Interoperability of heterogeneous devices is essential for the operation of the smart grid. Smart grid protocols must consider security [70, 71], routing efficiency [72, 73], device diversity, and quality of service requirements [74]. A review of routing, and in particular IPv6 Routing Protocol for Low Power and Lossy Networks (RPL) can be found in [75], and a review of Internet of Things (IoT) protocols is presented in [76] with a focus on its application to the smart grid.

2.1.3 Information System

The information system consists of the computing infrastructure at central and distributed nodes within the smart grid used for data management, optimisation and high-level network control, and the algorithms, platforms, services and software used within that infrastructure. Vast quantities of data are likely to be available from devices within the energy system, delivered through the communication system, and will require processing by the information system in order to implement smart grid monitoring and control. In order to realise the smart grid goals of improved monitoring and optimal grid operation the information system must implement computational paradigms that can handle the interoperability of large volumes of data such as distributed and cooperative processing. Listed below are components of the information system prominent in recent literature.

Distributed Computing: By distributing computing resources both data volumes and computing requirements at each processor can be reduced. Coordination of distributed computing resources is commonly through a multi-agent system (MAS) which relies on the communication subsystem to form links between agents [77, 78, 79, 80, 81]. Common control, optimisation and state estimation techniques employed by MAS include consensus [82, 31, 83, 39, 84, 30, 32, 85], particle swarm optimisation (PSO) [86, 87, 88], and game theory [89, 40, 90, 91, 92]. A review of multi-agent systems in microgrid applications can be found in [93].

Cloud Computing: Due to its ability to handle transmission, storage and processing of large volumes of data, cloud computing has been proposed as a major component of the smart grid information subsystem [16, 94]. Reviews of cloud computing application to the smart grid can be found in [95, 96].

Home Energy Management Systems (HEMS): HEMS provide infrastructure for the optimal management of energy sources and loads in a smart home environment [97, 98]. Benefits include the optimal management of power consumption through DG and appliance scheduling, real-time control, and demand response [99, 100, 101, 102]. A review of HEMS goals and applications can be found in [103].

Advanced Metering Infrastructure (AMI): AMI, or smart meters, replace traditional passive power meters and are capable of monitoring and reporting power usage and, in more advanced cases, power management through incorporation into a HEMS. AMI enable technologies such as load profile estimation and forecasting [104, 105, 106], load profile shaping through demand response [107], and are expected to reduce supplier costs through automation and improve demand side flexibility enabling a higher penetration of distributed renewable energy sources [108]. Challenges inherent in AMI include privacy and security [66, 67, 57], and energy theft [109, 110]. A review of AMI developments can be found in [111].

Active distribution network monitoring: Monitoring of distribution networks is minimal in most cases [12]. This presents a problem for many of the proposed smart grid technologies since system observability is required in order to obtain adequate input into the algorithms and processes, and use of pseudo measurements based on historical information is likely to be inadequate [19]. As such, additional monitoring in future smart distribution networks is expected in order to improve observability [20].

2.2 Distributed Smart Grid Optimisation and Control

Smart grids provide extensive opportunities to improve reliability, security, economics, efficiency, environmental impact, safety, and automation. However, achieving this potential requires intelligent management of potentially large volumes of data, especially as more DG and smart meters are rolled out. In this regard a centralised system meets limitations due to high data transmission and processing requirements, data synchronization and latency issues, and privacy concerns. In response to this, many distributed solutions to smart grid optimisation and control have been proposed. Distributed solutions aim to reduce centralized communication and processing burdens while maintaining privacy and optimal or near-optimal grid operation.

The focus of this thesis is on the distributed optimisation of smart grids. In particular the problems of network partitioning (Problem 1) and distributed optimal control (Problem 2) are addressed, and accordingly the following sections provide a brief review of the associated literature.

2.2.1 Network Partitioning

The structure of a traditional power system is hierarchical and extremely uni-directional: Power flows from large scale generator, through the transmission system into the distributions system before being delivered to consumers. However in the presence of increasing distributed generation (DG) a bi-directional system is required [13] – this is Problem 1 addressed in this thesis.

The increase of DG can have the benefit of improved network performance [14], however there is a limit to how much DG any bus within a network can support DG [15, 112], leading to potential voltage and fault current breaches [15]. In addition to DG, smart grids introduce complexity in the form of a range of heterogeneous components [16]. This motivates the need for more advanced regulation approaches for the smart grid, which requires a logical sub-structure to be applied. Numerous solutions to defining this logical structure have been proposed in the literature.

The following lists some of the relevant approaches to implementing a logical substructure to the smart grid commonly studied in recent research.

Virtual Power Plant (VPP) A VPP consists of multiple power sources logically connected to enable coordination, with the goal of reducing generation costs and increasing profits [9, 24]. VPP can provide ancillary services [21, 22], and assist in balancing demand with generation [24]. VPP operating models can take advantage of a liberalised market to aggregate distributed energy sources, irrespective of the underlying technologies, and provide an interface to the electricity market [113, 114]. This approach to aggregation allows for a service-centric model that accounts for uncertainty in resources availability, which is especially important when considering renewable resources [115, 116, 117].

Epsilon Decomposition Partitioning The process of epsilon decomposition operates on the sensitivity matrix and divides it into tightly and loosely coupled matrices [118]. By identifying tightly coupled buses, loosely couple buses can be ignored and regional calculations can be performed. Partitioned networks can then account for DG influence and regulate voltages to avoid breaching operating limits [119]. Due to the reliance on the sensitivity matrix which is inherently dynamic, the epsilon decomposition approach may meet limitations in the smart grid context where network state may fluctuate significantly.

Iterative Techniques Numerous techniques have been proposed that approximate optimal network partitioning based on a range of criteria. These approaches typically aim to form independent regions or microgrids that can operate independently. The concept of electrical distance is introduced in [120], which is based on the inverse of the sensitivity matrix, and aims to reduce the impact on the voltage at one bus due to a change in voltage at another. A genetic algorithm has been proposed with the objective of minimising energy exchange between microgrids [121]. Iterative partitioning techniques can also be used to identify optimal points for separation in an islanding strategy [122].

2.2.2 Distributed Control

The smart grid offers the potential for significant increases in optimal control through the intelligent utilisation of increased information. This information may be collected from a wide range of smart grid devices, then must be transmitted through the smart grid communication system and processed by the smart grid information system. The communication and processing of this data presents a problem for the centralised structure present in the traditional power system, which is limited in its scalability, especially as more DG and smart meters are rolled out [16, 17]. As such, distributed approaches have found favour in recent research. However, many distributed approaches presented within the literature do not accurately consider the power flow and regulatory constraints within smart distribution networks – this is Problem 2 addressed in this thesis.

The methods of distributed processing can be categorised into three main approaches: Independent regions (also referred to as zones or microgrids), centrally controlled regions, and cooperative regions without a central controller. The following expands on these approaches.

Independent Regions By taking advantage of network structure and electrical characteristics, approximations can be made by breaking smart grid problems into a simplified set of sub-problems or zones and allowing a distributed solution [123, 124]. This approach removes the central bottleneck by moving all processing to within the regions. For example, the sensitivity matrix can be decomposed through epsilon decomposition identifying loosely coupled zones which can be operated independently[119]. Once the regions are formed, local processing can be applied to voltage regulation [77], and to day-ahead scheduling for smart DG, storage and loads [92, 90, 91, 125, 87].

Through such region dividing algorithms, distributed solutions can be formulated that are highly scalable. However, the approximations required may be excessive in the context of a complex smart grid leading to sub-optimal solutions, and regions have the potential to compete with one another leading to oscillations. Furthermore, zone controllers must take into consideration information transmission delays from sensors [126], and errors in local estimations of state [127].

Centrally Coordinated Regions To overcome the issues present in a fully distributed solution varying levels of coordination can be utilised in order to improve estimates and optimality. In order to manage such coordination, each entity within the smart grid can be assigned an intelligent agent to form a multi-agent system (MAS), each agent of which can utilise the smart grid communication system to form links between agents [77, 78, 79, 80, 81]. One possible structure is the hierarchical MAS which utilises a central agent as a leader, keeping the central node's role separate to the roles of the distributed controllers. For example, optimisation can be performed by distributed agents in parallel while the leader plays a coordination role by updating global information such as Lagrange multipliers and aggregated load profiles [28, 26, 128, 125, 90, 92, 91].

The hierarchical MAS succeeds in distributing computational requirements but it does not remove the central bottleneck or fully alleviate communication burdens since all information exchange must pass through the central node. Communication burdens can be broken up by allowing inter-agent communication at lower levels of the MAS hierarchy. For example, the leader agent can drive the direction of the MAS in terms of an optimal operating point, while agents at lower levels of the hierarchy perform the optimisation in a distributed manner [87, 129]. This approach splits communication into transmission of optimisation related data and coordination signals, removing major regions of communication congestion.

Cooperative Regions A final improvement to the MAS communication structure is to remove the hierarchy altogether to form agents into a connected graph. Under this structure DG can cooperate through algorithms such as consensus to manage voltage regulation [77, 32, 33], economic dispatch [83], frequency control [130], and reactive power control [78, 27, 82], loads can be optimally scheduled or shed [89, 131, 132, 31], storage can be economically managed [39], and a range of heterogeneous smart grid agents can work together to conduct distributed control [133, 85, 84].

Considerations for the complete interdependence of buses and their associated agents within a smart distribution network has received limited attention in the literature. For example, the non-linear power flow constraints must be maintained and regulatory requirements such as nodal voltage limits must be observed [82, 84]. Such solutions can employ techniques including the alternating direction method of multipliers (ADMM) and consensus protocols in order to share information and distribute processing. However these issues are often not well addressed.

Distributed solutions to smart grid problems offer the scalability and flexibility required to support the future power supply system. By reducing the burden on the communication and information systems of the smart grid infrastructure, these solutions allow the energy system to perform its job optimally, securely, privately and reliably.

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CHAPTER 4

Constrained Coordinated Distributed Control of a Smart Grid with Asynchronous Information Exchange

Smart grid constrained optimal control is a complex issue due to the constant growth of grid complexity and the large volume of data available as input to smart device control. In this context, traditional centralized control paradigms may suffer in terms of the timeliness of optimization results due to the volume of data to be processed and the delayed asynchronous nature of the data transmission.

This chapter addresses thesis Problem 1 and the associated sub-problem of shifting optimal partition structure due to changes in network state over time. A partitioning algorithm is carefully designed through the combination of power flow analysis and epsilon decomposition in order to reduce estimation errors and improve optimality over a time window based on forecast outputs and loads. Thesis Problem 2 is also addressed through the presentation of a coordinated, distributed algorithm based on distributed, local controllers and a central coordinator for exchanging summarized global state information. The proposed model for exchanging global state information is resistant to fluctuations caused by the inherent interdependence between local controllers, and is robust to delays in information exchange. In addition, the algorithm features iterative refinement of local state estimations that is able to improve local controller ability to operate within network constraints. Application of the proposed coordinated, distributed algorithm through simulation shows its effectiveness in optimizing a global goal within a complex distribution system operating under constraints, while ensuring network operation stability under varying levels of information exchange delay, and with a range of network sizes.

4.1 Introduction

In recent years research into the broad field of smart grids has been extremely active. The exciting and powerful opportunities arising from new monitoring and controlling infrastructure has given rise to many new ideas and applications. However, the smart grid has also provided many new challenges, for example, the limitation of existing networks to accommodate new distributed generators (DG) [15, 112], and the added complexity from a wide range of heterogeneous components [16]. The research focus has included optimization of networks with high DG penetration [26, 42], optimization through direct control of storage and loads [134, 35, 135, 36, 37, 38], the application of the extensive smart grid monitoring and control devices for fault and breach management [43, 136], communication, data management and smart meters [10, 52, 137], the use of grid connected vehicles as both postponable loads and potential storage devices [47, 48, 45, 138, 46, 139], and optimal control of smart buildings [86, 99, 140, 101].

New methods are required to solve the range of optimization problems arising from this new and evolving environment. Many traditional methods employ centralized solutions which are limited in their ability to solve some of the larger and more complex problems presented by the smart grid. In particular their scalability, especially as more DG and smart meters are rolled out, increasing the demand on data transmission infrastructure and centralized computing resources [16, 17]. In such cases the volume of data and delays in the asynchronous data transmission may adversely affect the timeliness of centralized optimization results. As such distributed approaches are often beneficial.

Distributed approaches can utilize local data by partitioning the network according to such factors as the electrical properties of the network and the forecast power flow [1, 123, 124]. For example, in [119] epsilon decomposition is used to determine the range of influence of the network's DG, which is then utilized to control voltages should they exceed operating limits. Distributed approaches also benefit from the local optimization that is independent of a central bottleneck. For example in [77] a distributed approach is taken to voltage regulation utilizing the smart grid's set of intelligent and cooperative smart entities. And in [92, 90, 91, 125] a distributed game theoretic approach is taken to produce optimal day-ahead schedules for DG, storage and loads, and in [87] the optimal generation schedule for DG is evaluated through particle swarm optimization.

In spite of their benefits, many distributed approaches must make approximations in order to operate with either complete or partial independence from a central controller, leading to varying levels of sub-optimality.

Purely distributed solutions then aim to solve these sub-problems without the benefit of global state information or observation of changes in global state. Design of distributed algorithms must therefore be careful that these approximations don't lead to instability due to competing controls between neighbouring regions, and must take into consideration communication delays between sensors and local controllers [126], and inaccuracies in local estimates of state information [127]. Distributed solutions can therefore benefit greatly from some form of coordination in order to improve estimations.

The following list summarizes the key benefits of distributed, coordinated control:

- **Local data:** Utilization of local data reduces data size and improves privacy by reducing requirements for data sharing.
- **Local optimization:** Local controllers apply local data to their optimization routines which can improve the timeliness of optimization results.
- **Reduced Central Burden:** Computational requirements for the central controller are greatly reduced, even as the network dimension increases, since much of the burden is taken by the numerous local controllers.

In addition to these considerations, in the presence of controllable storage and postponable loads, optimization is no longer possible if only the current state is considered since actions taken in the present will affect future states and costs, resulting in the change of the original optimization problem. In this case, the optimization problem must consider the cost of operation into the future, and therefore a timely model is desirable which also considers the uncertainty introduced by the DGs, and to predict the future states based on the present state and future controls. Dynamic programming (DP) offers benefits over other methods in solving this type of problem due to its ability to reduce the problem's computational complexity by the combination of instantaneous decision making along the state trajectory and the optimal cost-to-go function associated with each state. In the case of a stochastic optimization problem, in particular problems where the expectation of future costs is difficult or impossible to calculate, approximate dynamic programming (ADP) can be applied to estimate the future costs. In addition to its ability to handle difficult stochastic problems, ADP has the added benefit of reducing a problem's dimensionality by summarizing the future states by a feature set.

ADP has been applied to many fields including control of the smart grid [141]. In [142] an optimal ADP algorithm is presented for the energy dispatch problem with grid-level storage, including a rigorous proof of the algorithm's convergence. The increased observability and controllability of the smart grid is utilized to apply a dual heuristic dynamic program to solving the dynamic stochastic optimal power flow (OPF) problem in [143]. Q-learning is applied to the optimal routing of shipboard power, storing discrete values for state-action value pairs in [144]. In [145] the problem of optimising DG output and storage is tackled by balancing supply and demand at the customer level through DP. In [146] operation of a micro-grid featuring both DG, heat supply and storage is optimised through DP. In [147] DP is applied to the multi-objective problem of optimally allocating DG to an existing network.

Application of ADP by power system operators has largely focused on the economic dispatch of power [141]. A review of the economic dispatch literature since 1990 is presented in [148]. These focus on resource allocation from the generation point of view and not the distribution system point of view. When applying ADP to the distribution system it is important to consider the added complexities of the network structure. Applying ADP to a distributed smart grid problem while considering the implications of reduced and delayed global state information exchange is the focus of this chapter.

In this chapter we present a coordinated, distributed algorithm based on distributed, local controllers and a central coordinator for exchanging summarized global state information, with the aim of optimizing resource allocation of DG and storage, and managing deterministic loads in the smart grid, while maintaining network operating constraints and allowing for delays in data exchange. The coordinated, distributed algorithm's objectives are to:

- Reduce the problem dimensionality compared to centralized methods,
- Improve local state estimation over purely distributed approaches,
- Be resistant to instability from competing local controllers, and
- Be robust in the presence of delayed information exchange.

This chapter is organized as follows: Section 4.2 provides the network analysis and dynamic programming framework on which the study's algorithms are built. Then in Section 4.3 our smart grid optimization problem is formulated as a distributed Optimal Power Flow problem. In Section 4.4 our proposed solution is presented through a centrally coordinated distributed approximate dynamic program with asynchronous information exchange between local and central nodes. Finally, a case study is presented illustrating the feasibility of this approach in Section 4.5, followed by the study's conclusion in Section 4.6.

4.2 Preliminaries

In this section we provide the relevant background to power flow analysis and dynamic programming, and the approximations to their solutions on which the distributed problem and solution of the chapter are built.

4.2.1 Approximate Power Flow

In preparation of the distributed problem formulation of Section 4.3.2 we seek approximations of the power flow equations. Power flow analysis of a network aims to find its steady-state operation, where network state is defined as bus power and voltage and line current. Newton-Raphson power flow analysis can calculate the network state given the bus admittance matrix and bus power for all busses. However this may not be possible if only a subset of the network's bus powers is known – such as in the case of a distributed optimization problem. In this case an approximation can be made.

From the Jacobian matrix of the Newton-Raphson power flow analysis the sensitivity matrix can be calculated:

$$\Lambda = \begin{bmatrix} \frac{\partial \delta}{\partial P} & \frac{\partial \delta}{\partial Q} \\ \frac{\partial |v|}{\partial P} & \frac{\partial |v|}{\partial Q} \end{bmatrix}.$$

The sensitivity matrix provides a linear approximation of the relationship between changes in nodal power and voltage as follows:

$$\begin{bmatrix} \delta_t \\ |v_t| \end{bmatrix} = \begin{bmatrix} \delta_0 \\ |v_0| \end{bmatrix} + \Lambda \begin{bmatrix} \Delta P_t \\ \Delta Q_t \end{bmatrix}, \tag{4.1}$$

where $|v_t|\angle\delta_t$ is the complex voltage at all busses at time t, and ΔP_t and ΔQ_t are vectors of the change in active and reactive power at all busses since time $t = 0$. Once these approximations are made, Λ no longer needs to be recalculated for each change in network state considered by the optimization process. This greatly reduces the burden on the distributed controller.

The period for which this approximation is appropriate will depend on the magnitude of any variations in network state. If significant changes occur in the network then the sensitivity matrix may require recalculation.

4.2.2 The Dynamic Smart Grid Problem: DP

A distributed approach to solving an optimization problem in the smart grid should aim to find the solution (or approximate solution) to the global problem. Here we present the global problem and dynamic programming approach that will be broken into a distributed problem in Section 4.3.1.

For an initial state x_0, the optimization problem is defined as follows:

$$\begin{aligned} &\text{Minimize } J_u(x_0) \\ &\text{Subject To } u = \{u_0, u_1, \ldots, u_{T-1}\} \\ &\qquad\qquad x_{t+1} = f_x(x_t, u_t, w_t), \\ &\qquad\qquad G(x_t, u_t) \leq 0, \ \forall \ t \in [0, T), \end{aligned} \tag{4.2}$$

where $J_u(\cdot)$ is the cost-to-go function to minimize, given state sequence $x = \{x_0, x_1, \ldots, x_T\}$ resulting from control sequence u. The receding prediction horizon, T, can be chosen such that the variance of the expected state of the system at $t = T$ is large, for example when forecast loads and available intermittent energy supplies are uncertain.

Given the dynamic nature of this problem, we apply the principals of dynamic programming (DP). DP selects the best decisions recursively from the last step backwards based on the cost of the present decision and the expected future cost. We define the cost-to-go recursively for a given control sequence $u = \{u_0, u_1, \ldots\}$:

$$J_u(x_t) = g(x_t, u_t) + \underset{w}{E}\left[J_u(x_{t+1}) | x_t, u_t\right], \tag{4.3}$$

where $g(x_t, u_t)$ is the cost of applying control u_t when in state x_t, and the expectation term $E\left[\cdot\right]$ is the expected future cost.

Dynamic programming aims to minimise J_u, that is, find the control sequence that solves

$$J(x_t) = \min_{u \in U_t(x_t)} \left\{ g(x_t, u) + \underset{w}{E}\left[J(x_{t+1}) | x_t, u\right] \right\}, \tag{4.4}$$

where $U_t(x_t)$ is the set of admissible controls when in state x_t and is governed by the inequality constraints, $G(x_t, u_t)$, of (4.2).

4.2.3 The Dynamic Smart Grid Solution: ADP

In preparation of the coordinated, distributed optimization approach of section 4.4, we seek an approximation of the expectation term in (4.4). Since state transitions are dependant on the previous state, action and random variables, the smart grid optimization problem may present a large number of reachable states for which the expectation of the future cost-to-go must be calculated. Specifically, computational requirements will grow exponentially with respect to the time horizon T. This is known as the “curse of dimensionality”. In the complex environment of the smart grid it is therefore appropriate to make some approximations.

As such, we replace the expectation from (4.4) with an approximation defined as $\tilde{V}_t(x_t)$:

$$\tilde{J}(x_t) = \min_{u \in U_t(x_t)} \left\{ g(x_t, u) + \tilde{V}_t(x_t^u) \right\} \tag{4.5}$$

where x_t^u is the post decision state at time t (i.e. the state after applying controls u but before applying the stochastic variations w_t [149]), and $\tilde{V}_t(\cdot)$ is the expectation approximation. We no longer need to calculate the cost an exponentially increasing number of times, however we do need to find an appropriate approximation model for $\tilde{V}_t(x_t^u)$ and find a process of training it. In Section 4.4.2 we present a distributed ADP algorithm that trains $\tilde{V}_t(\cdot)$ and approximates $\tilde{J}(x_t)$.

Below we discuss some considerations when choosing training sample paths and some convergence issues.

Policy Iteration: When dealing with high-dimensional problem spaces it can be difficult or impossible to evaluate all control policies that visit each state. As such a common solution to training the approximation (and the one used in this study) is to analyse a series of sample paths through Monte-Carlo simulation. Each sample path defines a control sequence $u^{(k)} = [u_0^{(k)}, \ldots, u_{T-1}^{(k)}]$ that is refined over a series of iterations (k). The sample paths can be chosen randomly forming an exploration policy. However, this approach can form a good approximation only if an appropriate representative sample set is taken from the state space. In other cases it may be possible to exploit the structure of the problem and follow an exploitation policy. If the sample paths are chosen according to a pure exploitation policy, then

$$u_t^{(k+1)} = \operatorname*{arg\,min}_{u \in U_t^{(k)}(x_t)} \left\{ g(x_t^{(k)}, u) + \tilde{V}_t(x_t^{u(k)}) \right\}, \tag{4.6}$$

where the choice of control at iteration $k+1$ is chosen according to the approximation of the optimum at iteration k. While some applications such as those studied in [141] can obtain optimal results from a pure exploitation policy, it is often required that a combination of exploration and exploitation be used to search for a broad approximation and then refine it.

ADP Convergence Issues: Approximate dynamic programming has been successfully applied to many applications. It is developed with an heuristic belief that if both the value function can be approximated with sufficient accuracy and optimal policies with respect to the approximated value function can be learnt, then the true optimal policy can be approximated with sufficient accuracy. Even though ADP is developed in this intuitive way, numerous proofs of both convergence and optimality have been developed for specific applications. Generally the nature of the approximation will determine the convergence and optimality of the ADP algorithm. According to [149], experimental results have shown the importance of the approximation's form being capable of capturing the true value function and new samples being able to improve the estimate of not only the sample state but also a large number of other states. In [150] a number of convergence results are reviewed for various continuous function approximations and in [141] and [142] the concavity of resource allocations is exploited to form a convergent algorithm.

4.3 The Distributed Smart Grid Problem

Here we formulate the distributed smart grid optimization problem as a distributed dynamic OPF problem. To this end the distributed dynamic smart grid problem and distributed approximate power flow is presented. This section then concludes with the calculation of voltage estimation errors as a measure of the limitation of a distributed approach.

4.3.1 Distributed Dynamic OPF

We consider a distribution network with sensitivities Λ, and featuring controllable DG and storage. The goal of dynamic OPF is to minimize costs $\sum_t g(x_t, u_t)$ over a time window $[0, T]$, by changing the control sequence $\{u_t\} \forall t \in [0, T]$, subject to state transition $x_{t+1} = f_x(x_t, u_t, w_t)$. We therefore define the cost-to-go according to (4.3) for control sequence $u = \{u_t\}$ recursively as

$$J_u(x_t) = g(x_t, u_t) + E_{w_t}\left[J_u(x_{t+1})|x_t, u_t\right], \tag{4.7}$$

where $J_u(x_t)$ represents the cost of network operation and power generation and, in the case of a deregulated competitive market, includes power import from third parties, and may also include bias towards renewable and distributed generation. Sequence $\{x_t\}$ and $\{u_t\}$ define the real and reactive power output and consumption of DG, loads, storage and smart devices, and storage capacities.

The vector of bus powers corresponding to generator busses is defined as S_{DG} and is constrained by minimum and maximum complex magnitudes

$$S_{DG}^- \leq |S_{DG}| \leq S_{DG}^+. \tag{4.8}$$

Similarly, bus powers corresponding to storage are defined as S_S and are constrained by

$$S_S^- \leq |S_S| \leq S_S^+. \tag{4.9}$$

The control vector is then defined as $u_t = [S_{DG,t}\ S_{S,t}]$. The vector of bus powers corresponding to load busses is defined as S_L, and the vector of storage capacities is defined as q and is subject to constraints

$$0 \leq q \leq q^+. \tag{4.10}$$

The control vector is then defined as $x_t = [S_{L,t}\ q_t]$. Finally we define the noise vector as $w_t = [\Delta S_{L,t}\ \Delta S_{DG,t}]$, where $\Delta S_{L,t}$ is a random variation in load power, and $\Delta S_{DG,t}$ is a random variation in DG output.

The network must be operated within the regulatory voltage limits specified by

$$\begin{aligned} \delta^- \leq \tilde{\delta}_t \leq \delta^+, \\ |v_t|^- \leq |\tilde{v}_t|_t \leq |v_t^+|, \end{aligned} \tag{4.11}$$

where the voltage approximations $[\tilde{\delta}_t\ |\tilde{v}_t|]^\top$ in (4.13) have be used. Constraints (4.8), (4.9), (4.10) and (5.3) together form the inequality constraints $G(x_t, u_t)$.

To present to the distributed dynamic OPF problem we assume that costs, controls and state are separable and can therefore be calculated by local controllers. Then given the subset of network busses B with a strong coupling to the local controller the problem is formally presented as follows:

$$\begin{aligned} &\min_{u_B} J_{u_B}(x_{B,0}) \\ \text{s.t.}& \\ &x_{B,t+1} = f_x(x_{B,t}, u_{B,t}, w_{B,t}), \\ &G_B(x_{B,t}, u_{B,t}) \leq 0,\ \forall\ t \in [0, T), \end{aligned} \tag{4.12}$$

where $u_B = \{u_{B,0}, u_{B,1}, \ldots, u_{B,T}\}$, $u_{B,t} \in u_t$, $x_{b,t} \in x_t$ and $w_{B,t} \in w_t$. An illustration of a network's subset structure is given in Figure 4.1. The solution to this distributed optimal power flow problem is defined in Section 4.4 where approximate dynamic programming is applied.

4.3.2 Distributed Power Flow

We denote the subset of busses known to the local controller as B, and all other busses in the network as $\sim B$. We can then say that changes in state in the busses of $\sim B$ will impact the state in the busses of B leading to estimate inaccuracies. We quantify this through the following linear approximation of (4.1):

$$\begin{aligned} \begin{bmatrix} \tilde{\delta}_t \\ |\tilde{v}_t| \end{bmatrix}_B &= \begin{bmatrix} \delta_0 \\ |v_0| \end{bmatrix}_B + \Lambda_{B,B} \begin{bmatrix} \Delta P_t \\ \Delta Q_t \end{bmatrix}_B + \Delta v_{B,\sim B} \\ \Delta v_{B,\sim B} &= \Lambda_{B,\sim B} \begin{bmatrix} \Delta P_t \\ \Delta Q_t \end{bmatrix}_{\sim B}. \end{aligned} \tag{4.13}$$

where $[\tilde{\delta}_t \; |\tilde{v}_t|]_B^\top$ are the approximate voltages in B, $\Lambda_{B,B}$ are the self-sensitivities within B, $\Lambda_{B,\sim B}$ are the sensitivities of busses in B with respect to external changes, $[\Delta P_t \Delta Q_t]_B^\top$ are the changes in power since time $t = 0$ at busses in B, and $[\Delta P_t \Delta Q_t]_{\sim B}^\top$ are the changes in active and reactive power since time $t = 0$ at all busses in $\sim B$.

The advantage of a distributed approach can be seen in (4.13). The subset of global state that is weakly coupled to the local controller is reduced to a single value, $\Delta v_{B,\sim B}$, which can be approximated as constant for the duration of the local controller's optimization. In Section 4.4.1 we present an algorithm based on ϵ decomposition to define the strongly coupled subset B based on the value of $\Delta v_{B,\sim B}$.

4.3.3 Distributed Voltage Approximation Error

The Sensitivity matrix (Λ_t) is time variant and we are approximating its value as constant as at time $t = 0$. We denote the error introduced at time t as e_t^Λ, which is dependant on the size of $[\Delta P_t \; \Delta Q_t]^\top$.

An error is also introduced due to the approximation of local voltages given in (4.13) due to a lack of real-time global state information. This error is quantified as the difference between the true change in voltage and the approximation given in (4.13):

$$\begin{aligned} e_t^v &= \begin{bmatrix} \delta_t \\ |v_t| \end{bmatrix}_B - \begin{bmatrix} \tilde{\delta}_t \\ |\tilde{v}_t| \end{bmatrix}_B \\ &= \Lambda_{B,\sim B} \begin{bmatrix} \Delta P_t \\ \Delta Q_t \end{bmatrix}_{\sim B} - \Delta \tilde{v}_{B,\sim B}, \end{aligned} \tag{4.14}$$

where $\Delta \tilde{v}_{B,\sim B}$ is the last known value of the change in voltages in B due to the network state external to B. Under normal operating conditions the sensitivities change slowly, justifying the linear approximation of (4.13). As such typically $e_t^\Lambda \ll e_t^v$, and so we concentrate on reducing e_t^v. This error represents a limitation to the distributed approach. As such the error is reduced through the iterative process between the global and local controllers described in Section 4.4.3.

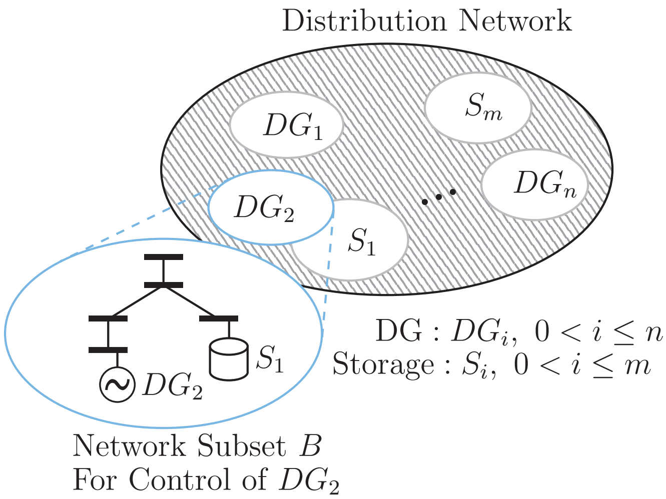

Figure 4.1: Distributed Network Structure

While e_t^{Λ} may be acceptably small while the changes in injected power vary minimally, however if the network state changes significantly then the central coordinator can recalculate and redistribute relevant portions of the sensitivity matrix. This process is further discussed in Section 4.4.3.

4.4 Proposed Coordinated Distributed Solution

A solution to the distributed problem presented in Section 4.3.1 is now offered as a coordinated, distributed, iteratively refined approximate dynamic program. To achieve this, the global problem must first be reduced to a set of distributed problems. This is achieved through a power flow based ϵ decomposition. The distributed problem is then solved through an ADP algorithm whose approximation of state and optimal control is refined through the introduction of a central coordinator.

4.4.1 Power Flow Based ϵ Decomposition

The following algorithm's objective is to define a set of strongly coupled buses, B, while minimizing voltage estimation errors at the controlled bus, $b \in B$, of a local controller. For the purpose of this study we assume that each controllable device in the smart grid has a local controller at its bus, which we designate as its controlled bus. The size of B is constrained such that both the communication and computation burdens at the local controller are reduced. This is achieved through observation of both the sensitivity matrix and forecast shifts in power and is based on ϵ decomposition (see [119] for an example of ϵ decomposition).

To minimize the impact of external state changes in the distributed power flow calculation of (4.13) and therefore reduce the error of (4.14) and improve state estimation, we must aim to minimize $\Delta v_{B,\sim B}$. As such we apply ϵ decomposition to the change in voltage at the controlled bus:

$$\Delta v_b = \Lambda_b \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \Lambda_{b,B} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}_B + \epsilon R \tag{4.15}$$

where R is a residual vector with all values less than 1, and ϵ is a scalar that quantifies the level of decoupling of subset B. We refer to the value of B that minimizes ϵR as the ϵ-tolerant subset.

Clearly $\epsilon R = \Delta v_{b,\sim B}$ from (4.13) and must be minimized across the time window of the optimization in order to find the best subset B. To this end we define the largest likely shift in active and reactive power from forecast data up to time T to be

$$\begin{bmatrix} \Delta P_{max} \\ \Delta Q_{max} \end{bmatrix} = \arg\max \left\{ \left\| \begin{matrix} \Delta P_t \\ \Delta Q_t \end{matrix} \right\|, t \in [0, T) \right\}, \tag{4.16}$$

where $\| \cdot \|$ denotes the vector's norm. Then the ϵ decomposition can be performed as a constrained minimization of $\Delta v_{B,\sim B}$:

$$\begin{aligned} B &= \underset{B \subset [1,n]}{\arg\min} \left\| \Lambda_{b,\sim B} \begin{bmatrix} \Delta P_{max} \\ \Delta Q_{max} \end{bmatrix}_{\sim B} \right\|, \\ C_B &\leq C_{(max)}, \end{aligned} \tag{4.17}$$

for an n bus network, where C_B is the number of controllable units in B, and $C_{(max)}$ is the maximum number of controllable units allowed for any local subset.

On a practical note, this minimization can be achieved with relative ease if we define the product of the sensitivity matrix and changes in power as an ordered sum. That is

$$\begin{bmatrix} \Delta \delta \\ \Delta |v| \end{bmatrix}_b = \Lambda_b \begin{bmatrix} \Delta P_{max} \\ \Delta Q_{max} \end{bmatrix} = \begin{bmatrix} \sum\limits_{i=1}^{n} \left(\frac{\partial \delta_b}{\partial P_i} \Delta P_i + \frac{\partial \delta_b}{\partial Q_i} \Delta Q_i \right) \\ \sum\limits_{i=1}^{n} \left(\frac{\partial |v_b|}{\partial P_i} \Delta P_i + \frac{\partial |v_b|}{\partial Q_i} \Delta Q_i \right) \end{bmatrix} \tag{4.18}$$

where P_i and Q_i are the elements of $[P_{max}\ Q_{max}]^\top$. We can then take the C_{max} most significant elements of the sum as our B and thereby the remaining summands make up the minimal $\|\Lambda_{b,\sim B}[\Delta P_{max}\ \Delta Q_{max}]^\top_{\sim B}\|$.

Through this process the size of $\Delta v_{B,\sim B}$ is reduced and therefore the likely local impact of changes external to the local controller are also reduced. The value of $\Delta v_{B,\sim B}$ is approximated as constant and further refined through information updates as described in Section 4.4.3.

Remark. The optimality of (4.12) is dependent on the error in state, which is defined by e^v, from (4.14). ϵ from (4.15) impacts the size of $\Delta v_{B,\sim B}$ and therefore the size e^v, and e^v determines the error in state since voltage $v \subset x$. Consequently ϵ will indicate the deviation from optimality in (4.12). Moreover, if e^v can be reduced, the approximation of optimality may also be improved.

4.4.2 Distributed Optimization Through ADP With Partial State Information

To solve the problem of (4.12) we must first be able to calculate the cost-to-go from (4.3), which involves a difficult to calculate expectation term. As such the expectation is replaced with an approximation defined as $\tilde{V}_t(x_t)$ and $J_u(\cdot)$ is approximated as follows:

$$\tilde{J}_u(x_t) = g(x_t, u) + \gamma_t \tilde{V}_t(x_t^u), \tag{4.19}$$

where x_t^u is the post decision state, and $\tilde{V}_t(\cdot)$ is the approximation of expected future costs. Assuming the estimator $\tilde{V}_t(\cdot)$ is available then the difficulty in applying (4.19) to solving (4.12) is only due to the dimensionality of u which has been reduced through the process described in Section 4.4.1.

Training of $\tilde{V}_t(\cdot)$ is performed according to the iterations of algorithm 1 by the local controller with controlled bus b, and ϵ tolerant subset B (the local controller of Figure 4.4.2 provides a simplified view of the process). Analysis of the presented algorithm reveals that the complexity of the ADP training is independent of total network size. To see this, consider the three most significant steps: The minimization of (4.20), the next state calculation of 2.5, and the sample calculations of 3.1. Assuming a quadratic cost function gives complexity of $g_B(\cdot)$ as $O(|B|^2)$, where $|B| = max\{|x|, |u|\}$ is the dimensionality of the local network subset. Given k samples at iteration k we assume that the complexity of the estimator is $O(k|B|)$. Then the complexity of each minimization step is $O(|B|^2) + O(k|B|)$, and the number of steps required is assumed to depend only on $|B|$. The next state function $f_x(\cdot)$ is assumed linear and therefore has complexity $O(|B|)$. Finally the sample calculations depend only on $g_B(\cdot)$ and therefore have complexity $O(|B|^2)$. The complexity of the algorithm therefore depends on the horizon T, iteration limit K, and network subset size $|B|$ which depends on the choice of C_{max} in (4.17), and does not depend on the total network size.

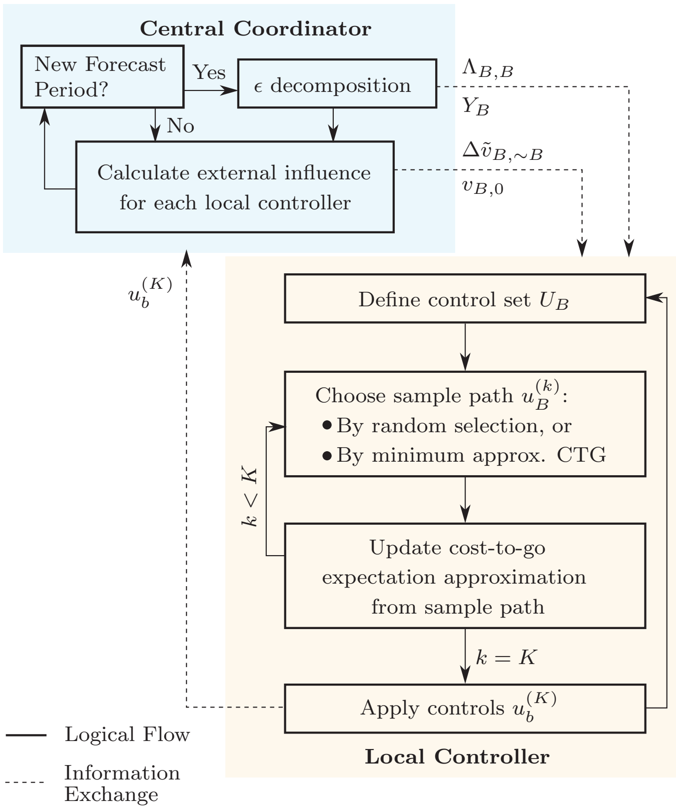

Figure 4.2: Central Iterations

Coordination Function Outputs $\Lambda_{B,B}$: Local sensitivities; Y_B: Local admittances; $\Delta\tilde{v}_{B,\sim B}$: External influences on voltage; $v_{B,0}$: Local voltages at time $t = 0$.
Local Controller Outputs k: Local iteration counter from 1 to K; U_B: Local admissible controls; $u_B^{(k)}$: Sample local control path; $u_b^{(K)}$: Approximation of optimal control at b.

Remark. The independence of algorithm 1 from the total network size allows the algorithm to be scaled to large networks while computational requirements can be tuned through parameters T, K, and C_{max}.

Remark. At step 2.2, in algorithm 1, $\rho^{(k)}$ is close to 0 for small values of k and close to 1 for large values of k. The choice of ρ will determine the rate of convergence, that is, how much the policy will explore the state-space before exploiting knowledge from the previous sample paths.

4.4.3 Refining Distributed Estimates Through Central Coordination

The solution from the algorithm of Section 4.4.2 relies on the approximation of voltage, based on $\Delta\tilde{v}_{B,\sim B}$. To improve this estimate, updated values of $\Delta\tilde{v}_{B,\sim B}$ are sent to the local controllers over a series of iterations by a central

Algorithm 1 ADP Training and Optimization for Network Subset B

1. Initialize current state $x_{B,0}$ and future cost estimators $\tilde{V}^{(0)}_{B,t}(\cdot)$, $\forall t \in [0, T)$, set k := 1.
2. For $t = 0, 1, ..., T-1$ calculate sample state trajectories $\{x_{B,t} | t \in [0, T)\}$:
   2.1 Choose a random control $\bar{u}_{B,t} \in U_{B,t}$.
   2.2 Choose an exploitation rate $\rho^{(k)}_t \in [0, 1]$.
   2.3 Find the approximate optimal control by solving
   $$\begin{aligned} u^{(k)}_{B,t} =& \rho^{(k)} \arg\min_{u \in U^{(k)}_{B,t}} (g_B(x^{(k)}_{B,t}, u) + \tilde{V}^{(k-1)}_{B,t}(x^{u(k)}_{B,t})) \\ &+ (1 - \rho^{(k)})\bar{u}_{B,t}. \end{aligned} \tag{4.20}$$
   2.4 Choose a random variation $w^{(k)}_{B,t}$.
   2.5 Next state: $x^{(k)}_{B,t+1} = f_x(x^{(k)}_{B,t}, u^{(k)}_{B,t}, w^{(k)}_{B,t})$.
3. Update the expected future cost estimator $\tilde{V}^{(k-1)}_{B,t}(\cdot)$:
   3.1 Sample costs-to-go: $y^{(k)}_{B,t} = g_B(x^{(k)}_{B,t}) + \gamma_t y^{(k)}_{B,t+1}$ assuming $y^{(k)}_{B,t} = 0 \; \forall t > T$.
   3.2 Update estimators: $(x^{(k)}_{B,t}, y^{(k)}_{B,t}), t \in [0, T]$.
4. $k := k + 1$. If $k \leq K$, for iteration limit K, go to 2.
5. Apply control $u^{(K)}_{b,0}$, for controlled bus $b \in B$, at time $t = 0$
6. Report $\{u^{(K)}_{b,t} | t \in [0, T)\}$ to the central coordinator.
7. Go to 2.

coordinator. The central coordinator is responsible for improving the local controllers' state estimation in order to reduce the likelihood of constraint breaches and to bring their solutions closer to optimal.

Such information exchange must be done with great care since it introduces a feedback loop in the global system. Specifically, oscillations may result from adjacent local controllers adjusting their controls in response to each other. To mitigate against this problem the controls are aggregated to form $u_t = \{u_{b,t} | \forall b\}$, for all controlled busses b, and are dampened by the introduction of a control step size $\alpha \in (0, 1]$ in the following iterative stochastic approximation:

$$\hat{u}_t^{(j)} = \alpha u_t + (1 - \alpha)\hat{u}_t^{(j-1)}, \tag{4.21}$$

where α is referred to as the step size since it dictates how far we update our j^{th} approximation of the optimal control, $\hat{u}^{(j)}$, in the direction of the new control policy, $u_t^{(j)}$.

Algorithm 2 describes the process for exchanging updated external voltage approximations with the local controllers, utilizing the dampened control values specified by (4.21) (the central coordinator of Figure 4.4.2 provides a simplified view of the process). The algorithm can handle delayed information exchange by simply assigning $[P_t^{(j)} \; Q_t^{(j)}]_B := [P_t^{(j-1)} \; Q_t^{(j-1)}]_B$, at step 4., when new information is not available at central iteration (j) from subset B. This will have the effect of slowing down convergence, but so long as new information is received regularly the convergence argument of Section 4.4.4 holds.

At each iteration of algorithm 2 the network state is assessed (refer to step 2.) and if required the local approximations $\Lambda_{B,B}$ are updated (see Section 4.3.3 for a discussion of the error due to the constant Λ approximation). This update to the sensitivity matrix is performed according to the aggregated network controls defined by (4.22) for the present time. The relevant portions of the sensitivity matrix, $\Lambda_{B,B}$, are then distributed to the local controllers who use the updated matrix for subsequent calculations. In this way the linear approximation of power flow through time invariant Λ can be adapted to the state and model drifting.

Remark. The control variables of algorithm 1 are continuous with respect to time, as such the algorithm approximates optimal control as constant for any given time step. However, each central iteration of algorithm 2 will trigger optimal values to be updated by the local controller and so the control update rate is dependent only on the central coordinator's update rate. This brief period allows for regular corrections to the optimal control in response to system state changes which are assumed minimal within the update period.

Remark. At step 5.3, in algorithm 2, $\Delta v_{B,\sim B}$ summarizes the state of the loosely coupled network busses with respect to local controller B and is therefore able to reduce the required information exchange between the central coordinator and local controllers.

Algorithm 2 Central Coordination of Information Updates

1. Initialize admittances Y, sensitivities, Λ, optimal control estimates $\hat{u}_t^{(0)} = 0,\ \forall t \in [t_0, T)$, and set j := 1.

2. If $[\Delta P_{max}\ \Delta Q_{max}]^\top$ has changed (ref. (4.16)):

   2.1 Define ϵ tolerant subsets according to (4.17).

   2.2 Send updated information to each local controller, B: $\Lambda_{B,B}$ and Y_B.

3. Aggregate controls for all n locally controlled busses:

$$u_t = \{u_{b,t}^{(j)} | \forall b \in [1, n]\}. \tag{4.22}$$

4. Update optimal control estimates according to (4.21).

5. Update local controller voltage information:

   5.1 Obtain power changes: $[\Delta P_t^{(j)}\ \Delta Q_t^{(j)}]^\top \subset \hat{u}^{(j)}$.

   5.2 Calculate the external voltage changes for each local controller, B:

$$\Delta\tilde{v}_{B,\sim B} := \Lambda_{B,\sim B}[\Delta P_t^{(j)}\ \Delta Q_t^{(j)}]^\top_{\sim B}. \tag{4.23}$$

   5.3 Send updates to local controllers: $v_{B,0}$, $\Delta\tilde{v}_{B,\sim B}$.

6. Let $j := j + 1$. Go to 2..

4.4.4 The Convergence of Dampened Information Exchange

Here we provide an heuristic explanation of the convergence resulting from the appropriate selection of the step size α. Consider a network under steady state operation that experiences a change in controls by local controller B. Let us define the change in controls at B at iteration j as

$$\Delta u_B^{(j)} = u_B^{(j)} - u_B^{(j-1)} \tag{4.24}$$

and the impact of this change on the voltage of the remaining busses in the network as

$$\Delta v_{\sim B,B}^{(j)} = \Lambda_{\sim B,B} \begin{bmatrix} \Delta P^{(j)} \\ \Delta Q^{(j)} \end{bmatrix}_B, \tag{4.25}$$

where $[\Delta P^{(j)}\ \Delta Q^{(j)}]^\top_B \in u_B^{(j)}$. From (4.25) we can see that the changes in voltage at external busses has a linear relationship with the change in power resulting from the change in control at B. As such we assume

$$\left|\Delta v_{\sim B,B}^{(j-1)}\right| < \theta \left|\Delta u_{B}^{(j-1)}\right|, \ \forall\ j > 1, \tag{4.26}$$

for constant $\theta \in (0, \infty)$. We further assume, based on (4.13), that the change in controls $\Delta u_{\sim B,B}^{(j)}$ in response to the change in voltage, $\Delta v_{\sim B,B}^{(j-1)}$, is also linearly constrained. As such we assume

$$\left|\Delta u_{\sim B,B}^{(j)}\right| < \phi \left|\Delta v_{\sim B,B}^{(j-1)}\right|, \ \forall\ j > 1, \tag{4.27}$$

for constant $\phi \in (0, \infty)$. Finally we assume that corresponding constraints exist for changes in control external to B influencing the voltage and control at B such that

$$\left|\Delta v_{B,\sim B}^{(j)}\right| < \theta \left|\Delta u_{\sim B,B}^{(j)}\right|, \ \forall\ j > 0, \tag{4.28}$$

and

$$\left|\Delta u_{B,\sim B}^{(j+1)}\right| < \phi \left|\Delta v_{B,\sim B}^{(j)}\right|, \ \forall\ j > 0. \tag{4.29}$$

The constant θ represents the limit of the network's response to a local change in control, and ϕ the limit of the local control adjustment to a local change in voltage. As such, for assumptions (4.26) to (4.29) to hold we must assume that the variation of Λ is bounded for all $j > 0$ since constants θ and ϕ are dependant on Λ which is in fact a function of voltage according to the partial derivatives of the power flow equations. This assumption is reasonable while the network operates within voltage constraints according to (4.12).

Given that the true changes in control are $\alpha\Delta u$, and given assumptions (4.26) to (4.29), then

$$\begin{aligned} \left|\Delta u_{B,\sim B}^{(j+1)}\right| &< \phi \left|\Delta v_{B,\sim B}^{(j)}\right| \\ &< \alpha\phi\theta \left|\Delta u_{\sim B,B}^{(j)}\right| \\ &< \alpha\phi^2\theta \left|\Delta v_{\sim B,B}^{(j-1)}\right| \\ &< \alpha^2\phi^2\theta^2 \left|\Delta u_{B}^{(j-1)}\right|. \end{aligned} \tag{4.30}$$

Given that Δu is in fact a random variable further assumptions must be placed on α. We assume that α is non-negative, $\sum_{t=0}^{\infty} \alpha = \infty$ and $\sum_{t=0}^{\infty} \alpha^2 < \infty$. Then, given a choice of α that satisfies $\alpha^2\phi^2\theta^2 < 1$, it follows that as $j \to \infty$, $u^{(j+1)} - u^{(j)} \to 0$ and the network will again return to steady state operation.

4.5 Case Study

We consider the case of an operator controlling DG and storage in a distribution network with the aim of minimizing power import into the network from third party suppliers. Tests were conducted on networks with a range of sizes, and optimization was achieved through control of both DG and storage, and for the sake of a simpler presentation only the constraints of (4.8), (4.9), (4.10) and the voltage magnitude constraints of (5.3) were applied. Formally, we aimed to approximately solve (4.2) with $J(\cdot)$ approximated by (4.19), and

$$g(x_t, u_t) = |S_{0,t}| \operatorname{sgn}(\mathrm{Re}\{S_{0,t}\}), \tag{4.31}$$

where $S_{0,t}$ is the complex power at time t and at bus 0, with the slack bus assumed to be at index 0 with respect to voltage and power vectors v_t and S_t, and admittance matrix Y. The future cost-to-go approximation defined as $\tilde{V}_t(x_t)$ in (4.19) was implemented through Kernel Regression applied with a Gaussian Kernel [151].

4.5.1 Scenarios

Coordinated, distributed optimization was applied to both a small scale network and a series of randomly generated networks of varying sizes. Experiments on the small scale network were aimed at verifying the coordinated, distributed algorithm's ability to perform comparably with centralized approaches in terms of optimality and state estimation, and to assess the algorithm's convergence with information exchange delays. The larger network experiments aimed to assess the coordinated, distributed algorithm's scalability in terms of convergence and processing time.

The small scale tests were performed on a network based on the IEEE 13 node test feeder network [152] featuring both DG and storage. Distributed generators were connected to busses 611, 645, 646, 675, 680 and 684, with total maximum output potential greater than the network's total peak load. Storage was connected to busses 632, 645, 671 and 684 each with a 1MWh capacity.

The large scale tests used randomly generated networks, featuring similar operating conditions to the IEEE 13 node test feeder network, featuring DG, storage and stochastic loads at similar densities and capacities.

Forecast DG output and demand curves used by all scenarios are presented in Figure 4.3.

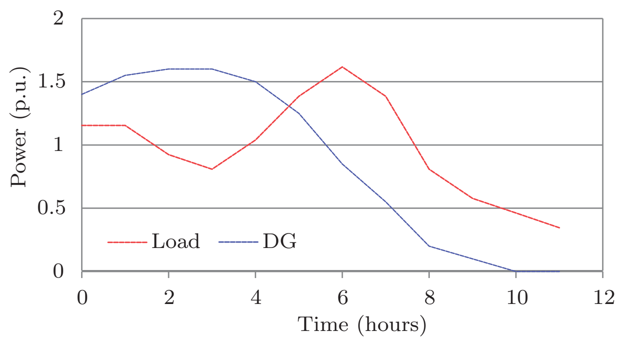

Figure 4.3: 12 Hour Forecast

4.5.2 Localization

Applying the ϵ decomposition of Section 4.4.1 resulted in the local controller ϵ-tolerant subsets as described in Table 4.1. For the problem specified by

Table 4.1: Network ϵ Decomposition

Controlled Device	ϵ-Tolerant Subsets (B)
DG 611	611, 634, 645, 684
DG 645	634, 645, 646
DG 646	634, 645, 646
DG 675	634, 645, 675, 680
DG 680	634, 645, 675, 680
DG 684	611, 634, 645, 684
Storage 632	632, 634, 645, 675
Storage 645	634, 645
Storage 671	634, 645, 671, 675
Storage 684	634, 645, 684

(4.31) to be solved by each subset, the local version of the cost function must first be defined according to the distributed OPF problem of (4.12) and the distributed ADP problem of (4.20). The local cost contribution is derived by calculating the changes in power imported into the distribution network due to changes in the busses of subset B. The total power import can be given as

$$\begin{aligned} S_{0,t}^* &= v_{0,t}^* Y_0 v_t, \\ &= v_{0,t}^* Y_0 (v_0 + \Delta v_t), \end{aligned} \tag{4.32}$$

where Y_0 is the admittance matrix row corresponding to the slack bus, and Δv_t is derived from $\Lambda[\Delta P_t \; \Delta Q_t]^\top$ with $[P_t \; Q_t] \subset (u_t \cup w_t)$. Given that many terms in (4.32) are constant and assuming the slack voltage is 1 p.u., the minimization can be given as

$$\min_{u_t} |S_{0,t}| = \min_{u_t} |Y_0 \Delta v_t|. \tag{4.33}$$

This can then be applied to the local changes in subset B to give the local cost function:

$$g_B(x_t, u_t) = |Y_0 \Delta v_{\cdot,B,t}| \operatorname{sgn}(\operatorname{Re}\{Y_0 \Delta v_{\cdot,B,t}\}), \tag{4.34}$$

where $\Delta v_{\cdot,B,t}$ are the changes in complex voltage due only to changes in control in subset B.

4.5.3 Results

Here we present the results of the simulations. The following demonstrations illustrate the coordinated, distributed optimization algorithm's ability to maintain costs compared to a centralized approach, to maintain voltages without full network state information, to be stable under delayed information exchange, and to maintain performance with increasing numbers of local controllers. Numerous executions of the simulation were performed to ensure that the results presented here are a representative set for the average case.

Centralized, Distributed and Coordinated Cost Comparison: The scenario was deliberately selected such that a centralized comparison could be made. Here we present the minimized costs according to four optimization approaches:

1. A deterministic dynamic program using expected values for random variables,
2. The ADP algorithm of Section 4.4.2 applied in a centralized manner to the entire network,
3. The coordinated, distributed algorithm,
4. The average from a series of random control sequences used for relative comparison.

The coordinated, distributed results have been taken after a number of iterations once the algorithm has stabilized. The results depicted in Figure 4.4 show that although there has been a drastic reduction in state information (refer to Table 4.1), the coordinated, distributed algorithm is able to provide a good approximation of the optimal solution.

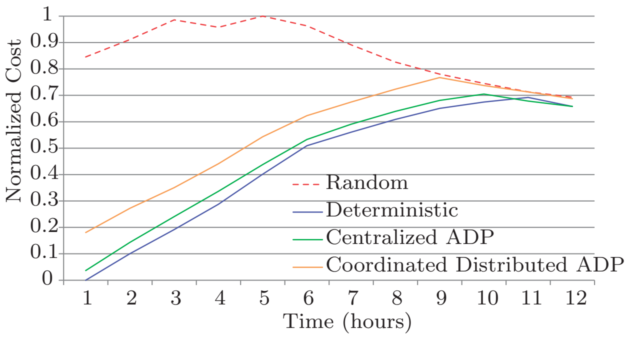

Figure 4.4: Cost Comparison of Optimization Methods

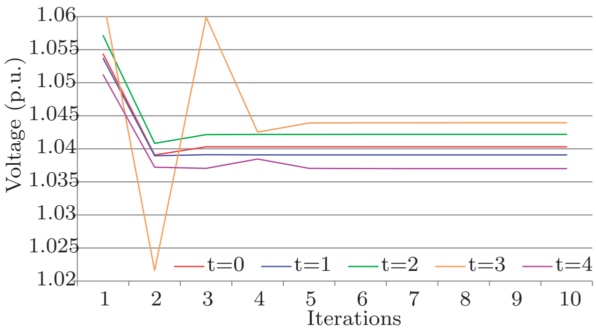

Figure 4.5: Maximum Voltage at Each Iteration

Voltages: As discussed in Section 4.3.3, due to local controllers possessing only a subset of the full network's state, voltage calculations are approximate only. This raises the possibility of underestimating voltages and subsequently approximating optimal controls that lead to voltage breaches according to $G(x_t, u_t)$ in Section 4.3.3. Here we demonstrate the ability of the central coordination to reduce the chance of such breaches. Figure 5.3 shows the maximum network voltages at each iteration for times $t = 0, 1, 2, 3, 4$ (other times did not exhibit voltages breaches for any iteration). At iteration 1, when local controllers have no global state information, locally optimal controls results in voltage breaches. At iteration 2, after dampened information has been shared, each local controller overreacts, drastically reducing the voltage. Subsequent iterations result in a stabilization of the voltages within the voltage magnitude constraints of (5.3).

Information Exchange Delays: We simulate the case of delayed data transfer between local controllers and the central coordinator by associating a probability, p_u, with each local controller which determines if the updated local state information is made available. For example a probability of $p_u = 0.5$

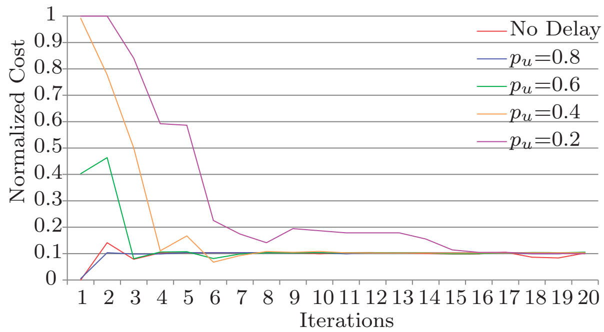

Figure 4.6: Cost Convergence With Information Exchange Delays

represents the case where, on average, each local controller has updated data available only every second central iteration. The coordinated, distributed optimization was performed for varying values for the data transfer probability and is presented in Figure 4.6. The information delay behaves as a type of dampening. For minor delays, such as where there is only a 20% chance of information delay ($p_u = 0.8$), the controls and therefore cost converges quickly. As the value of p_u decreases the system takes longer to converge. However, it is clear that even when updates are received from local controllers only 20% of the time, the algorithm is still able to converge. Another side effect of the dampening effect of the delay is that optimization with delayed information may be less prone to overshoot. For example, if the case of no delay is compared with the case of $p_u = 0.8$, then it can be seen that the delayed case has less overshoot and in fact converges more quickly.

Dampened Updates: To illustrate the importance of dampening the selected controls between central iterations in (4.21), the centrally calculated costs are compared over iterations with no dampening and dampened with a step size of $\alpha = 0.8$. Results can be seen in Figure 4.7. The issues discussed in Section 4.4.4 can clearly be seen, with the undampened case exhibiting oscillations. In addition to the oscillation in control and cost in the undampened case, the voltage estimates are unable to stabilize and so they switch between over- and under-estimating. This results in breaches on every second central iteration. On the other hand, the dampened control case stabilizes quickly.

Scalability: In order to test the scalability of the coordinated, distributed algorithm, it was applied to a range of randomly generated networks of varying sizes. The simulations were performed on a quad core Pentium i5 with 16GB of RAM running Windows 7. Table 4.2 lists the algorithm's processes and average execution times. The ϵ decomposition has the longest processing

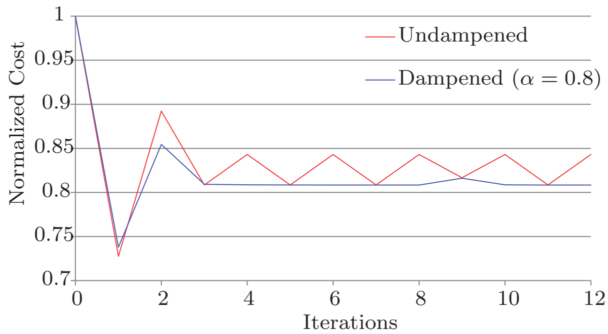

Figure 4.7: Effects of Dampening Controls of Cost

Table 4.2: Process Times

Equation	**Process**	**Time (ms)**
(4.17)	ϵ Decomposition	692
(4.20)	ADP Training and Optimization	99.5
(4.22)	Control Aggregation	0.037
(4.23)	Voltage Change Updates	2.61

time, however it is not performed frequently.

Figure 4.8 presents the processing time required for ADP training and optimization by the local controller, and for voltage change updates made by the central coordinator. The timing samples were taken across 20 central iterations and give the average time taken for each task per iteration. Voltage change updates show that there is a quadratic increase in processing time as the number of local controllers increases. This is due to the calculation of (4.23). However, the time taken for these updates is significantly shorter than the time taken for ADP training and optimization.

ADP training and optimization results in Figure 4.8 show that the local optimizations exhibit constant time processing regardless of the number of local controllers. These results are consistent with the complexity analysis performed in Section 4.4.2, which showed that, since there is a bound on the number of controllable devices within each local controller's network subset, network size does not impact the processing requirements of the ADP training and optimization algorithm as it would in the centralized case.

The series of test cases presented in Figure4.8 were also assessed for convergence. A subset of results were selected for time $t = 0$ and are presented in Figure 4.9. The cost-to-go curves show good convergence after fewer than 20 central iterations for the range of number of local controllers tested, and

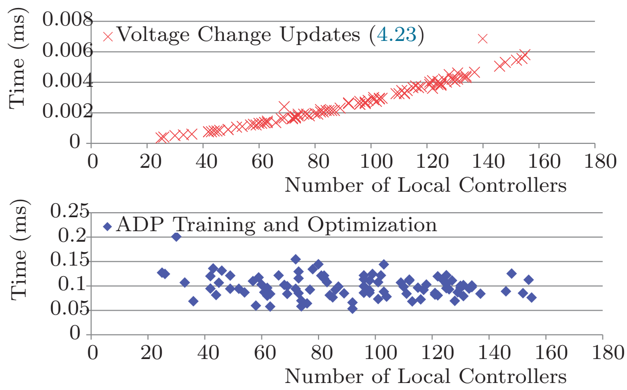

Figure 4.8: Process Duration For a Single Central Iteration

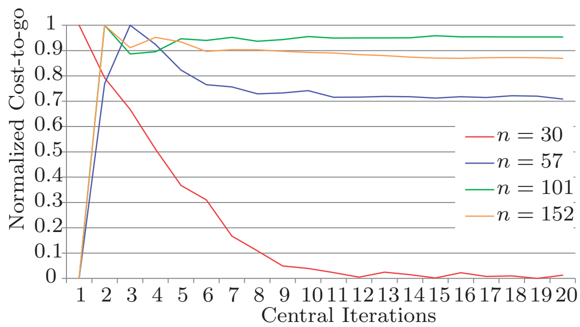

Figure 4.9: Cost-to-go Convergence for Networks with n Local Controllers

the presented cases are representative of all experimental results.

These results suggest that the coordinated, distributed algorithm can easily handle many local controllers. This is an important feature of the algorithm when considering the case of a more powerful coordinating server and numerous low powered local controllers.

4.6 Conclusion

We have presented a coordinated, distributed, constrained optimization algorithm for regulating smart grid technologies that utilizes the well established methods of approximate dynamic programming and optimal power flow. Our algorithm carefully summarizes global state information through ϵ decomposition such that local controllers can improve their approximation of optimal control without being overburdened by the high-dimensional state of the entire distribution network. Additionally, the reduced state information is updated over a series of iterations controlled by a central coordinator, providing local controllers with continually improved estimations and allowing for asynchronous global information exchange.

The proposed coordinated, distributed algorithm features reduced dimensionality reducing calculation complexity and as such can be applied to online optimization, even in the case of low powered distributed controllers. Complexity analysis of the local optimization algorithm has shown that it is independent of total network size, and as such the proposed distributed optimization approach is scalable to large networks. The centralized nature of the algorithm's coordination allows it to operate in an asynchronous manner making it robust to communication delays, and the flexibility of the algorithm allows it to be adapted to the costs and constraints specific to the needs of the smart grid operator.

Through our case study simulation we have demonstrated how the use of a subset of network information can lead to an approximately optimal solution. We have also demonstrated the coordinated, distributed algorithm's ability to improve local state estimation of voltages and to perform well with respect to cost minimisation when compared to centralized solutions. Also, we have shown that even in the presence of asynchronous information exchange, the coordinated, distributed approach can converge to a near optimal solution. Finally, we have demonstrated the algorithm's ability to scale well with respect to the number of local controllers.

CHAPTER 5

Smart Grid Optimization Through Asynchronous, Distributed Primal Dual Iterations

The diversity of components in the smart grid and issues such as scalability, stability and privacy have led to the desire for more distributed control paradigms. In this chapter thesis Problem 2 is addressed by considering the problem of optimizing smart grid operation with separable global costs and separable but non-convex constraints, while considering important aspects of network operation such as power flow and nodal voltage constraints. A localized primal dual method is applied through the use of an augmented Lagrange function which is used to overcome the issues of non-convexity in the presence of non-linear equality constraints. The non-separability of the augmented Lagrange penalty function is addressed through use of local and neighbourhood communication leading to a completely distributed solution of the global problem.

5.1 Introduction

Rapid growth in smart grid technologies in the smart distribution system has led to the need for new approaches in monitoring and control. For example, the increase in available information from monitoring devices has led to new data communication and processing requirements, and to the potential for privacy issues. And the increase in the controllability and observability of smart grid components such as distributed generators, storage and controllable demands, offers great potential for the improvement of network stability and optimality. These issues motivate the need for updated optimization algorithms, and in particular decentralized adaptations of problem formulations such that data communication volumes can be reduced, data processing can be distributed, privacy can be protected and system control can maintain optimal or near-optimal operation.

With heterogeneous power generators, diverse local objectives, and limited communication bandwidth, distributed control becomes more desirable in smart grid systems. Distributed approaches typically take advantage of network structure to simplify solutions and communications. For example, in [119] the sensitivity matrix is decomposed into regions which can each operate independently. The drawback of such an approach is that the regional division requires approximations of the state of the remainder of the network and algorithms must be careful to avoid oscillations due to competition between controllers. A similar solution is to provide an hierarchical communication and control structure such that the roles of the central entity (e.g. the utility) are separated from the roles of the distributed controllers. This is an example of a multi-agent system (MAS), where, in this case, the central coordinator has a leadership role. For example, in [28, 26, 128], optimization is performed in parallel by each agent while a central coordinator updates global information such as Lagrange multipliers and aggregated load profiles. And in [125, 90, 92, 91] a game theoretic approach is taken with a leader who is responsible for calculating and communicating aggregated global information. The hierarchical approach often succeeds in distributing the computation of the optimization problem, however local controllers are typically required to solve a full minimization problem at each iteration, and communication issues still exist due to the central coordinator acting as a hub for all information exchange.

An alternative and perhaps favoured form of a MAS is to provide a system structure that allows for inter-agent communication. This approach can greatly reduce the communication burdens by removing the central bottleneck of the coordinator. For example, in [87], particle swarm optimization is employed to manage price bidding in the day-ahead market, where bids are placed with an auctioneer. The optimization takes place through communication between a range of agents, with the auctioneer agent coordinating the bidding process. And in [129] a consensus protocol is developed to estimate global information at the local controller level, while a leader agent drives the consensus variable in the direction of a globally optimal solution. This distributed approach through a MAS greatly reduces centralized computation and communication. However, many such systems still rely on some form of centralized coordination, which may be undesirable since it provides a central point of failure.

Recent research has focused on purely distributed approaches, such as consensus-based algorithms, which are able to remove the need for a leader in the MAS. That said, many algorithms still choose to elect a leader agent which is responsible for driving the solution according to a global objective. Purely distributed algorithms (without central leadership) are presented in [77, 32] where distributed generators act as agents and cooperate for the purpose of voltage regulation. And in [89], where load scheduling is performed based on real-time pricing through cooperation between consumers. Incremental cost algorithms are presented in [39, 31, 83] where the economic dispatch problem is solved through consensus-based estimation of the dual variables relating to inseparable global constraints. A similar solution through the primal-dual perturbation method is presented in [85]. These distributed approaches present significant improvements in terms of their ability to solve smart grid optimization problems in a purely distributed manner without central points of communication or processing. The complete interdependence of nodes in a distribution network, especially in a heavily meshed network warrants the extension of such approaches to the case of maintaining non-linear power flow constraints, and observing regulatory requirements such as nodal voltage limits.

Few examples exist of purely distributed smart grid optimization algorithms which account for these implications of network structure and regulatory constraints. In [82] a region based approach is taken where each agent is responsible for controlling reactive power output under power flow constraints within its region. The optimal solution is found through a fully distributed form of the alternating direction method of multipliers (ADMM) involving repeated local minimizations. While the presented algorithm solves the OPF problem without any centralized coordination or control, it requires synchronization between agents and solves a simplified problem based on convexity assumptions and linear approximation of constraints. And in [84] a completely distributed, consensus-based solution to optimal power flow is presented by commissioning each node to complete a full minimization across not only its own state variables but its neighbours also.

Ideally an optimization algorithm will operate without the need for a central coordinator to manage communication or to drive convergence, and with only simple communication and computational requirements placed on the local controllers, and consider global issues such as power flow which would be built into the distributed computation. In this chapter we address the problem of optimizing smart grid operation with separable global costs and separable but non-convex constraints, while considering important aspects of network operation such as power flow and nodal voltage constraints. A localized primal dual method is applied through the use of an augmented Lagrange function which is used to overcome the issues of non-convexity in the presence of non-linear equality constraints. The non-separability of the augmented Lagrange penalty function is addressed through use of local and neighbourhood communication leading to a completely distributed solution that is able to reach a global optimum. The solution is first carefully formed in a centralized manner such that a global optimum can be found through an iterative process. This centralized form is then shown to be equivalent to the distributed form due to the presented solution's structure, without requiring any modifications to the algorithm. The resulting algorithm is straightforward, fully distributed and capable of producing a solution to the global problem. Each iteration is not required to reach the optimum of the distributed optimization sub-problem. Even though that can simplify the analysis, it is computationally more efficient to compute only a finite number of steps for the distributed optimization. The algorithm is then extended to the asynchronous case.

The remainder of the chapter is organized as follows: The power flow constrained problem is presented in Section 5.2. The solution via the augmented Lagrange function is then given with discussions of convergence, optimality and feasibility in Section 5.3, which is then extended into a distributed form in Section 5.4. Finally simulation test results are presented in Section 5.5 demonstrating the algorithm's convergence and feasibility, and Section 5.6 concludes the presentation.

5.2 Problem Formulation

The power distribution network is modelled as the undirected graph (N, Y). Here N is the set of all busses in the network with $0 \in N$ representing the slack bus, and complex $|N| \times |N|$ matrix Y is the bus admittance matrix defining the edges of the graph. The network consists of dispatchable distributed generators (DG) identified by the set $N_G \subset N$, and non-generation busses $N_L \subset N$. Without loss of generality we assume $N_L = N \backslash N_G$. The objective of smart grid optimization is then to control DG power output to minimize generation cost (or equivalently maximize generation utility in the case of customer generation) within network operating constraints.

Dispatchable DG operational costs are a function of network state x, and are given by the bounded convex function

$$c(x) = \sum_{i \in N_G} c_i(p_i, q_i), \tag{5.1}$$

where $p_i, q_i, i \in N_G$ are the active and reactive nodal powers, and operate according to the DG operational constraints

$$p_i \in [p_i^-, p_i^+], q_i \in [q_i^-, q_i^+], \ \forall\, i \in N_G. \tag{5.2}$$

The set of bus powers corresponding to non-generator busses, that is $\{p_i, q_i : i \in N_L\}$, are considered to be uncontrollable, but observable, elements of network state. The combined vectors of real and reactive bus powers are defined as $p = [p_0 \ldots p_N]$ and $q = [q_0 \ldots q_N]$ respectively. Slack bus powers, p_0 and q_0, are uncontrolled and operate in the typical manner supplying or sinking current to ensure zero net power flow within the network.

Bus voltages are modelled by their real and reactive parts $e = [e_0 \ldots e_N]$ and $f = [f_0 \ldots f_N]$ respectively, with slack bus voltages set to $e_0 = 1$ and $f_0 = 0$, and all other voltages constrained according to

$$|[e_i \ f_i]| \in [v^-, v^+], \ \forall i \in N \backslash \{0\}, \tag{5.3}$$

where $|\cdot|$ denotes the magnitude of the complex voltage. The active and reactive representation of voltage is used instead of the more common magnitude and angle representation due to its simplification of derivatives, which will be of benefit during development of the optimization algorithm.

Bus voltages are related to bus powers via the real and reactive power flow constraints

$$\begin{aligned}
g_p(x) &= [g_{p_1}(x) \ldots g_{p_N}(x)]^\top = \mathbf{0}, \\
g_q(x) &= [g_{q_1}(x) \ldots g_{q_N}(x)]^\top = \mathbf{0}, \\
g_{p_i}(x) &= \sum_{j \in N} (e_i e_j G_{ij} + f_i f_j G_{ij} + f_i e_j B_{ij} - e_i f_j B_{ij}) - p_i, \\
g_{q_i}(x) &= \sum_{j \in N} (f_i e_j G_{ij} - e_i f_j G_{ij} - e_i e_j B_{ij} - f_i f_j B_{ij}) - q_i,
\end{aligned} \tag{5.4}$$

where we define the network state as the power and voltage at each bus such that $x = [p\ q\ e\ f]^\top$. The problem can now be formally presented as follows:

$$\begin{aligned} &\text{Minimize } c(x) \\ &\text{Subject to } g_p(x) = \mathbf{0}, \\ &\qquad\qquad\quad g_q(x) = \mathbf{0}, \\ &\qquad\qquad\quad x \in X, \end{aligned} \tag{5.5}$$

where $c(x)$ is convex according to (6.1), $g(x) = [g_p(x)\ g_q(x)]^\top$ is a non-convex function, and X is the closed convex set of admissible states defined according to inequality constraints (5.2) and (5.3). The OPF problem, such as that presented in (5.5), is known to be nonlinear, non-convex, and NP-hard in general [153], and solutions typically guarantee only a local optimum. Discussions and numerical solutions to centrally solving OPF problems have been presented in the literature. Please see the review paper of OPF methods [154]. Rather than adding another recipe in this line, the objective of this chapter is to formulate a solution that can easily be computed in a fully distributed manner. For the remainder of the presentation it is assumed that a solution to problem (5.5) exists.

5.3 Augmented Lagrangian Optimization

We seek a local minimum of (5.5) to which end we consider the augmented Lagrange function

$$\begin{aligned} L(x,\lambda) =& c(x) + \lambda_p^\top g_p(x) + \lambda_q^\top g_q(x) \\ &+ \tfrac{\alpha^{(k)}}{2}(\|g_p(x)\|^2 + \|g_q(x)\|^2), x \in X, \end{aligned} \tag{5.6}$$

where $\lambda = [\lambda_p\ \lambda_q]$ is the vector of Lagrange multipliers corresponding to the real and reactive power flow constraints respectively, and $\{\alpha^{(k)}\}$ is an increasing sequence of penalty multipliers which, as it increases, ensures that the minimum of (6.7) with respect to x approaches a feasible solution in terms of the power flow constraints (5.4).

To find a local minimum of (6.7) we apply the method of multipliers which produces the sequence $\{x^{(k)}\}$ according to the following iterative procedure

$$x^{(k+1)} = \arg\min_{x \in X} L(x, \lambda^{(k)}). \tag{5.7}$$

The following proposition can be found in [155] and is included for the convenience of the reader.

Proposition 5.3.1. Given a bounded sequence $\{\lambda^{(k)}\}$, and sequence $\{\alpha^{(k)}\}$ such that $0 < \alpha^{(k)} < \alpha^{(k+1)}$ for all k and $\alpha^{(k)} \to \infty$, and compact, isolated set X^* of local minima of problem (5.5), then the sequence $\{x^{(k)}\}$ produced by local solutions of (6.8) contains a subsequence $\{x^{(k)}\}_K$ that converges to the local minimum $x^* \in X^*$.

Proof. See [155] proposition 2.2. $\square$

For values of $x^{(k)}$ produced by (6.8) that lie on the interior of X, we can say that the gradient satisfies

$$\|\nabla_x L(x^{(k+1)}, \lambda^{(k)})\| = 0. \tag{5.8}$$

However calculating the exact minimum is often not practical. In fact, while $x^{(k)}$ is an interior point of X, (6.9) can be slackened and an approximation to the minimum can be found such that

$$\|\nabla_x L(x^{(k+1)}, \lambda^{(k)})\| \leq \epsilon^{(k+1)}, \tag{5.9}$$

where the sequence $\{\epsilon^{(k)}\}$ satisfies $\epsilon^{(k)} \geq 0$ for all k and $\epsilon^{(k)} \to 0$ ([155] proposition 2.3). In the case that a point $x^{(k)}$ lies on the boundary of X condition (9) may not be achievable, in which case (7) must be applied directly.

According to the discussion above, the gradient projection method [156], subject to (6.10), is applied to approximately solving the minimization (6.8) and is followed by an optimal multiplier estimate update seen below in (5.12).

For each $i \in N_G$ optimal power estimations are updated:

$$\begin{aligned} p_i^{(k+1)} &:= P_X\left\{p_i^{(k)} - \gamma^{(k)}\frac{\partial L(x^{(k)}, \lambda^{(k)})}{\partial p_i}\right\}, \\ q_i^{(k+1)} &:= P_X\left\{q_i^{(k)} - \gamma^{(k)}\frac{\partial L(x^{(k)}, \lambda^{(k)})}{\partial q_i}\right\}, \end{aligned} \tag{5.10}$$

and for each $i \in N\backslash\{0\}$ voltage estimations are updated:

$$\begin{aligned} e_i^{(k+1)} &:= P_X\left\{e_i^{(k)} - \gamma^{(k)}\frac{\partial L(x^{(k)}, \lambda^{(k)})}{\partial e_i}\right\}, \\ f_i^{(k+1)} &:= P_X\left\{f_i^{(k)} - \gamma^{(k)}\frac{\partial L(x^{(k)}, \lambda^{(k)})}{\partial f_i}\right\}, \end{aligned} \tag{5.11}$$

where $P_X\{\cdot\}$ is projection on X, and the step size $\gamma^{(k)}$ is chosen to ensure a reduction in $L(\cdot)$; this can be achieved by performing a line search, an algorithm for which is provided in Section 5.4.2. Updates (5.10) and (5.11) are repeated until condition (6.10) is met, or, if $x^{(k)}$ lies on the boundary of X, until two consecutive Lagrange function values L and L', calculated by (6.7), satisfy $|L - L'| \leq \delta_L$, where δ_L is the target precision.

For each $i \in N \backslash \{0\}$ multiplier estimations are then updated:

$$\begin{aligned} \lambda_{p_i}^{(k+1)} &= P_\Lambda \left\{ \lambda_{p_i}^{(k)} + \alpha^{(k)} g_{p_i}(x^{(k+1)}) \right\}, \\ \lambda_{q_i}^{(k+1)} &= P_\Lambda \left\{ \lambda_{q_i}^{(k)} + \alpha^{(k)} g_{q_i}(x^{(k+1)}) \right\}, \end{aligned} \tag{5.12}$$

where $P_\Lambda\{\cdot\}$ is projection on $\Lambda = \{\lambda : |\lambda| \leq \lambda^+\}$ given some positive constant λ^+, and ensures the boundedness of $\{\lambda^{(k)}\}$. The choice of λ^+ does not impact the results of proposition 5.3.1 since no assumptions are made on $\{\lambda^{(k)}\}$ aside from boundedness, however it may impact the rate of convergence and, if possible, should be chosen such that $|\lambda^*| \ll \lambda^+$, where λ^* is the optimal Lagrange multiplier. Although λ^* is not known in advance, if x^* lies in the interior of X then

$$\nabla_x c(x^*) + \nabla_x g(x^*)\lambda^* = 0, \tag{5.13}$$

and if $\nabla_x g(x^*)$ has rank $2|N|$ then

$$\lambda^* = -\left(\nabla_x g(x^*)^\top \nabla_x g(x^*)\right)^{-1} \nabla_x g(x^*)^\top \nabla_x c(x^*). \tag{5.14}$$

Given that $x^* \in X$ we have

$$\lambda^* \in \{-\left(\nabla_x g(x)^\top \nabla_x g(x)\right)^{-1} \nabla_x g(x)^\top \nabla_x c(x) : x \in X\}, \tag{5.15}$$

the bounds of which can be analysed according to the range of $g(x)$ and $c(x)$ enforced by the constraint set X in order to estimate an appropriate value for λ^+. If necessary, this analysis can be supported by assessing a representative range of preliminary simulation results.

The Lagrange multiplier updates of (5.12) can be seen as a gradient ascent step, or an increase in the penalty for non-zero $g(x)$. According to proposition 5.3.1 convergence to a solution of (5.5) can be achieved for any bounded sequence $\{\lambda^{(k)}\}$ so long as $\alpha \to \infty$. However, by applying the updates of (5.12), this result can be improved upon such that for large enough $\alpha^{(k)}$ the multiplier estimates $\lambda^{(k)} \to \lambda^*$. Specifically, if x^* is an interior point of X and $\nabla_{xx}^2 L(x^*, \lambda^*) > 0$, then for a given $\lambda^{(k)}$ there exist positive scalars $\alpha_\lambda^{(k)}$ and M such that if $\alpha^{(k)} \geq \alpha_\lambda^{(k)}$ the distance of $\lambda^{(k+1)}$ from λ^* is bounded according to $\|\lambda^{(k+1)} - \lambda^*\| \leq M\|\lambda^{(k)} - \lambda^*\|/\alpha^{(k)}$ ([155] proposition 2.4). If these conditions cannot be met, for example if x^* lies on the boundary of X, then the boundedness of $\lambda^{(k)}$ and increase of $\alpha^{(k)}$ ensure convergence according to proposition 5.3.1.

5.4 Distributed Solution

The preparation of the iterative solution presented in the preceding section was designed such that it could easily be implemented in a fully distributed

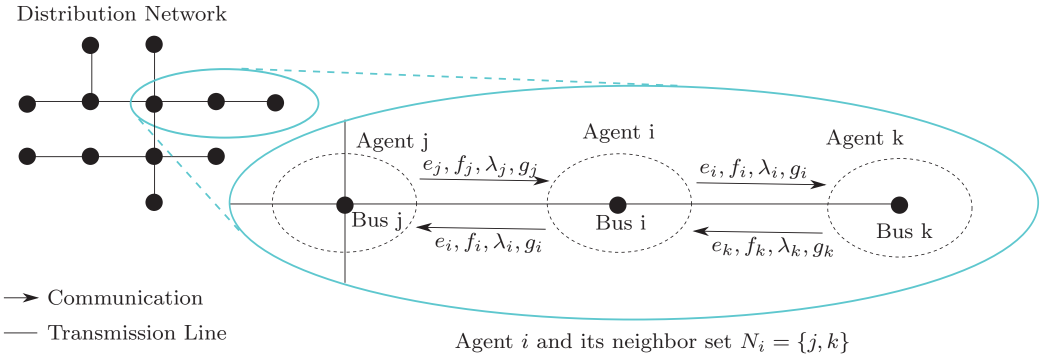

Figure 5.1: Example of inter-agent communication: Each bus within the distribution network is equipped with an agent which communicates with its neighbours.

manner. Recall the power network structure defined by the graph (N, Y). The distributed implementation relies on communication between nodes in the set N, with the set $N_i \subset N$ representing the set of neighbours to bus i such that $Y_{ij} \neq 0$ iff $j \in N_i$. To this end the communication network structure is chosen to duplicate the power network structure, making it ideal for communication technologies such as power line communication [157] – recall that $Y_{ij} \neq 0$ also represents a power line connection between busses i and j. Given this communication structure and given that $Y_{ij} = 0$ when busses i and j are not neighbours, all partial derivatives required by (5.10) and (5.11), and all data required for dual variable updates (5.12) can be expressed in local terms for agent i:

$$p_i, q_i, e_i, f_i, \lambda_{p_i}, \lambda_{q_i}, g_{p_i}, g_{q_i}$$

and in neighbourhood terms for all $j \in N_i$

$$e_j, f_j, \lambda_{p_j}, \lambda_{q_j}, g_{p_j}, g_{q_j}.$$

Note that $g_{p_i}, g_{q_i}, g_{p_j}$ and g_{q_j} represent the latest values of the respective power flow constraints. An illustration of the nodal communications is presented in Figure6.1. The partial derivatives required for (5.10) and (5.11) are given below, with matrices G and B representing the real and imaginary parts of the admittance matrix Y respectively.

$$\frac{\partial L(x,\lambda)}{\partial p_n} = \frac{\partial c_n(p_n, q_n)}{\partial p_n} - \lambda_{p_n} - \alpha g_{p_n}(x), \tag{5.16}$$

$$\frac{\partial L(x,\lambda)}{\partial q_n} = \frac{\partial c_n(p_n, q_n)}{\partial q_n} - \lambda_{q_n} - \alpha g_{q_n}(x), \tag{5.17}$$

$$
\begin{aligned}
&\frac{\partial L(x,\lambda)}{\partial e_n} = \\
&\quad \sum_{i\in N_n} \lambda_{p_i} \left(e_i G_{in} + f_i B_{in}\right) + \sum_{i\in N_n} \lambda_{q_i} \left(f_i G_{in} - e_i B_{in}\right) \\
&\quad + \lambda_{p_n} \left(\sum_{j\in N_n} \left(e_j G_{nj} - f_j B_{nj}\right) + 2G_{nn} e_n \right) \\
&\quad + \lambda_{q_n} \left(\sum_{j\in N_n} \left(-f_j G_{nj} - e_j B_{nj}\right) - 2B_{nn} e_n \right) \\
&\quad + \alpha \sum_{i\in N_n} \left(g_{p_i}(e_i G_{in} + f_i B_{in}) + g_{q_i}(f_i G_{in} - e_i B_{in})\right) \\
&\quad + \alpha g_{p_n} \left(\sum_{j\in N_n} (e_j G_{nj} - f_j B_{nj}) + 2e_n G_{nn} \right) \\
&\quad + \alpha g_{q_n} \left(\sum_{j\in N_n} (-f_j G_{nj} - e_j B_{nj}) - 2e_n B_{nn} \right),
\end{aligned}
\tag{5.18}
$$

$$
\begin{aligned}
&\frac{\partial L(x,\lambda)}{\partial f_n} = \\
&\quad \sum_{i\in N_n} \lambda_{p_i} \left(f_i G_{in} - e_i B_{in}\right) + \sum_{i\in N_n} \lambda_{q_i} \left(-e_i G_{in} - f_i B_{in}\right) \\
&\quad + \lambda_{p_n} \left(\sum_{j\in N_n} \left(f_j G_{nj} + e_j B_{nj}\right) + 2G_{nn} f_n \right) \\
&\quad + \lambda_{q_n} \left(\sum_{j\in N_n} \left(e_j G_{nj} - f_j B_{nj}\right) - 2B_{nn} f_n \right) \\
&\quad + \alpha \sum_{i\in N_n} \left(g_{p_i}(f_i G_{in} - e_i B_{in}) + g_{q_i}(-e_i G_{in} - f_i B_{in})\right) \\
&\quad + \alpha g_{p_n} \left(\sum_{j\in N_n} (f_j G_{nj} + e_j B_{nj}) + 2f_n G_{nn} \right) \\
&\quad + \alpha g_{q_n} \left(\sum_{j\in N_n} (e_j G_{nj} - f_j B_{nj}) - 2f_n B_{nn} \right).
\end{aligned}
\tag{5.19}
$$

Clearly all information required for each iteration is available from either the local bus or the neighbouring busses. As such the synchronous form of the distributed algorithm is equivalent to the central solution provided by iterations (5.10), (5.11) and (5.12).

5.4.1 Synchronous Distributed Algorithm

Here we consider a synchronous implementation where each agent performs a single update of the primal and dual variables in parallel at each iteration. The synchronous distributed algorithm is summarized in algorithm 3.

Algorithm 3 Synchronous Distributed Algorithm

Initialize state variables $[p^{(0)}\ q^{(0)}] = \arg\min\{c(x)\}$, $e^{(0)} = \mathbf{1}$, $f^{(0)} = \mathbf{0}$, $x^{(0)} = [p^{(0)}\ q^{(0)}\ e^{(0)}\ f^{(0)}]^\top$, and dual variables $\lambda^{(0)} = [\lambda_p^{(0)}\ \lambda_q^{(0)}]^\top$.
For $k := 1$ **to** K**:**

1. For each $i \in N\backslash\{0\}$ update primal variables:

   1.1 Compute $x_i^{(k+1)}$ from (5.10) and (5.11) subject to (6.10).

2. For each $i \in N\backslash\{0\}$ update multipliers:

   2.1 Compute $\lambda_i^{(k+1)}$ from (5.12).

3. Choose next penalty multiplier $\alpha^{(k+1)} > \alpha^{(k)}$.

While this approach has the advantage of being iteration-by-iteration equivalent to the centralized approach defined by (5.10), (5.11) and (5.12), it is difficult to implement in practice since it still requires some form or central coordination for the timing of iterations and the updates of α and ϵ, and the distributed choice of γ cannot be calculated easily through a backtracking line search since the line search algorithm requires calculation of $L(x, \lambda)$. First we address the issue of γ selection through a distributed form of backtracking line search.

Remark. In algorithm 3 step 3. the penalty multiplier is updated to ensure an increasing sequence (refer to Proposition 5.3.1). This is most commonly achieved through selection of a parameter $\rho > 0$ and setting $\alpha^{(k+1)} = \rho\alpha^{(k)}$, for example [158].

5.4.2 Distributed Backtracking Line Search

Selecting an appropriate step size γ at each iteration is essential for the success of algorithm 3 since as $\alpha \to \infty$ the step size must compensate for the resultant increase in gradient to avoid over-stepping. The backtracking line search algorithm can easily be used centrally by choosing γ such that

$$L(x - \gamma\Delta x, \lambda) < L(x, \lambda) - \delta\gamma\Delta x^\top \Delta x, \tag{5.20}$$

where $\Delta x = \nabla_x L(x, \lambda)$ is the gradient descent step direction, and $\delta \in (0, 1)$. This guarantees an improvement at each gradient descent step. However, calculating the value of the augmented Lagrangian requires central knowledge that is not available to the agent. Instead, given local step direction $\Delta x^n = \nabla_{x_n} L(x, \lambda)$ with $\Delta x_i^n = 0, \ \forall i \neq n$ for agent n, the augmented Lagrangian defined at $x - \gamma \Delta x^n$ can be separated into two parts as follows

$$L(x - \gamma \Delta x^n, \lambda) = L(x, \lambda) + \Delta L(x - \gamma \Delta x^n, \lambda). \tag{5.21}$$

We denote $\Delta L(x - \gamma \Delta x^n, \lambda) = \Delta c + \lambda^\top \Delta g + \frac{\alpha}{2} \Delta g^2$ for convenience and see that it can be calculated through local and neighbourhood information by considering its components:

$$\begin{aligned} \Delta c &= c(x - \gamma \Delta x^n) - c(x), \\ \Delta g &= g(x - \gamma \Delta x^n) - g(x), \\ \Delta g^2 &= \|g(x - \gamma \Delta x^n)\|^2 - \|g(x)\|^2. \end{aligned} \tag{5.22}$$

Clearly Δc can be calculated with only local information. Expanding out all terms of $g(x - \gamma \Delta x^n)$ reveals that Δg requires only local and neighbouring terms since $\Delta x_i^n = 0, i \neq n$. For example,

$$\begin{aligned} \lambda_p^\top \Delta g_p &= \lambda_{p_n} \Delta g_{p_n} + \sum_{j \in N_n} \lambda_{p_j} \Delta g_{p_j}, \\ \Delta g_{p_n} &= (2 e_n \Delta e_n + \Delta e_n^2 + 2 f_n \Delta f_n^2) G_{nn} - \Delta p_n \\ &+ \sum_{j \in N_n} \left((e_j \Delta e_n + \Delta f_n f_j) G_{nj} + (\Delta f_n e j - \Delta e_n f_j) B_{nj} \right), \\ \Delta g_{p_j} &= (e_j \Delta e_n + f_j \Delta f_n) G_{nj} + (\Delta f_j e_n - \Delta e_j f_n) B_{nj}. \end{aligned} \tag{5.23}$$

Finally Δg^2 can be defined in terms of $g(x)$ and Δg. For example

$$\begin{aligned} [g_{p_i}(x - \gamma \Delta x^n)]^2 &= [g_{p_i}(x) + \Delta g_{p_i}]^2 \\ &= [g_{p_i}(x)]^2 + [\Delta g_{p_i}]^2 + 2 g_{p_i}(x) \Delta g_{p_i}, \\ \therefore \Delta g_{p_i}^2 &= \Delta g_{p_i} \left(\Delta g_{p_i} + 2 g_{p_i}(x) \right). \end{aligned} \tag{5.24}$$

Distributed backtracking line search is then given according to algorithm 4.

5.4.3 Asynchronous Distributed Algorithm

To present an asynchronous implementation of algorithm 3 the principals of stochastic gradient decent are applied where only the local step direction is considered at each iteration. Each agent operates without waiting for neighbours to have updated values, but rather utilizes the most recent information.

Algorithm 4 Distributed Backtracking Line Search

Initialize: Select constants $\delta \in (0,1)$ and $\beta \in (0,1)$, and set $\gamma := 1$.
While $\Delta L(x - \gamma\Delta x, \lambda) < -\delta\gamma\Delta x^\top \Delta x$**:**

$\gamma := \beta\gamma.$

This process can be expressed as an update of a random subset of the state variables at each iteration. Specifically, (5.10) and (5.11) are replaced at each iteration by

$$x^{(k+1)} = x^{(k)} - \gamma^{(k)}\Delta x^{n(k)}, n \subset N\backslash\{0\}, \tag{5.25}$$

where $\Delta x^{n(k)}$ is the gradient decent step direction for the random selection of agents n, such that $\Delta x_i^{n(k)} = 0,\ i \notin n$. It is reasonable to assume that the expected direction of such an updating process, over a number of iterations, will be close to the true gradient direction. More specifically, for $\hat{x} = \arg\min_x L(x, \lambda)$, we can write

$$\mathbf{E}_n[(\Delta x^{n(k)})^2] \leq A(x^{(k)} - \hat{x})^2,\ A \geq 0, \tag{5.26}$$

which simply describes the fact that the step direction variance decreases to zero as $x^{(k)} \to \hat{x}$, and is met when the eigenvalues of the Hessian matrix of $L(x, \lambda)$ are bounded. Under condition (5.26) the iterations (5.25) converge to $\hat{x}$ almost surely. The reader is referred to [159] and the references therein for a more detailed convergence discussion.

In addition to storing local state, agent i maintains local values $\alpha_i > 1$ and $\epsilon_i \geq 0$ such that $\alpha_i \to \infty$ and $\epsilon_i \to 0$. The local admissible controls according to (5.2) and (5.3) are defined as X_i. The resulting algorithm is presented in algorithm 5.

Remark. At each iteration of algorithm 5 a random subset of agents is chosen to update. This is equivalent to all agents asynchronously updating in parallel such that within the time period between iterations k and $k+1$ only the subset $n \subset N\backslash\{0\}$ have completed their updates.

5.5 Simulation Results

We consider the case of a heavily distributed multi-agent system (MAS); that is, a system where there are many, potentially light weight, controllers and sensors (agents) that rely on local neighbourhood communications for optimization but not control. Simulations were performed on a 35 bus subset of the IEEE 123 node test feeder system [152]. Specifically, buses 1 to 33

Algorithm 5 Asynchronous Distributed Algorithm

Initialize state variables $[p_i^{(0)} \; q_i^{(0)}] = \arg\min\{c_i(x_i)\}$, $e_i^{(0)} = 1$, $f_i^{(0)} = 0$, $x_i^{(0)} = [p_i^{(0)} \; q_i^{(0)} \; e_i^{(0)} \; f_i^{(0)}]^\top$, and dual variables $\lambda_i^{(0)} = [\lambda_{p_i}^{(0)} \; \lambda_{q_i}^{(0)}]^\top$, $\forall i \in N \backslash \{0\}$.
For $k := 1$ **to** K**:**

1. Select random subset $n \subset N \backslash \{0\}$.
2. For each $i \in n$ update primal and dual variables:
   2.1 Compute $x_i^{(k+1)}$ from (5.10) and (5.11) with $\gamma^{(k)}$ selected from algorithm 4 and subject to (6.10).
   2.2 Compute $\lambda_i^{(k+1)}$ from (5.12).
   2.3 Choose next penalty multiplier $\alpha_i^{(k+1)} > \alpha_i^{(k)}$.
3. For each $i \notin n$ carry over all variables:
   3.1 $x_i^{(k+1)} = x_i^{(k)}$.
   3.2 $\lambda_i^{(k+1)} = \lambda_i^{(k)}$.
   3.3 $\alpha_i^{(k+1)} = \alpha_i^{(k)}$.

were considered, with bus 149 connected to the substation modelled as a slack bus regulating voltage to 120V. DG with output capacity of 500kW and reactive power capability were attached to buses 1, 3, 8, 14, 18, 26 and 29, and were tasked with optimal voltage control based on local quadratic cost functions $c_i(p_i, q_i) = a_i p_i^2 + b_i q_i^2$, through the application of algorithm 5. The base voltage was set to 1p.u.= 120V, voltage constraints were set to a magnitude of [0.95, 1.05] p.u., and base power was set to 1p.u.= 10kVA.

Figure 5.2 presents a comparison between the Lagrange function (6.7) and the cost function (6.1) during the iterations of algorithm 5. It is clear that the Lagrange and cost functions converge which implies that $g(x) \to 0$ and a feasible solution according to equality constraints has been found. The feasibility of the solution with regards to inequality constraints is evident in Figure5.3 where voltages can be seen to be constrained within the voltage limits. In the presented scenario bus 16 has its voltage inequality constraints activated, setting its voltage magnitude to 0.95 p.u.. This can be seen in Figure5.3 by the bus voltage marker sitting on the 0.95 p.u. boundary.

The progression of the gradients over iterations can be seen in Figure 5.4. As mentioned above, bus 16 has had its voltage inequality constraint acti-

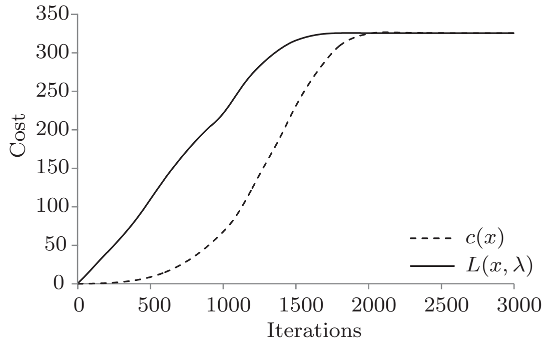

Figure 5.2: Convergence of Lagrange function $L(x, \lambda)$ to cost function $c(x)$ over iterations of algorithm 5

vated, meaning that the unconstrained optimum lies outside of the set X. As such the gradient of bus 16 has been neglected from Figure 5.4. It is evident that the gradients converge to zero (with respect to busses that have not had inequality constraints activated), satisfying (6.10). In this instance a sequence $\{\epsilon^{(k)}\}$ could be defined such that (5.10) and (5.11) required only a single step each iteration.

Convergence of the estimates of optimal real and reactive DG power outputs is presented in Figure 5.5. Each line represents the solution of a single DG equipped agent. The solution can be seen to converge after approximately 2000 iterations, each iteration requiring only a very small amount of processing and communication from each controller. The presented simulation is in some aspects a worse case scenario in terms of convergence rate since it is unlikely that the initial state will be so far from the optimal state; in a typical scenario it is fair to assume that the initial state would be set to the current system state or the state from the previous algorithm execution instead of fixed values such as $e = 1$ and $f = 0$.

Processing time requirements for each agent over iterations was analysed and the results are presented in Figure 5.6. The times are the average processing time taken per agent at each iteration. Overall, the average processing time is very short per iteration due to the local controllers only performing a minimization step rather than a full minimization. The hyphenated line shows the average processing time per agent over all iterations. Processing time increases very slowly as the optimal solution is reached. This is due to the backtracking algorithm taking longer to find an appropriate step size since near the optimal solution steps must be very small in order to continue to reduce the cost without 'over-stepping' the minimum.

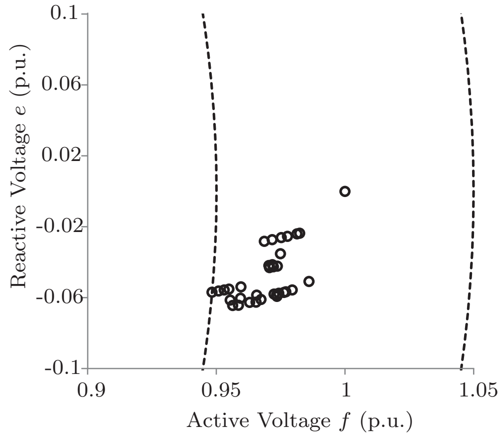

Figure 5.3: Nodal voltages after optimization through algorithm 5; voltage magnitudes are constrained between [0.95 1.05] p.u.

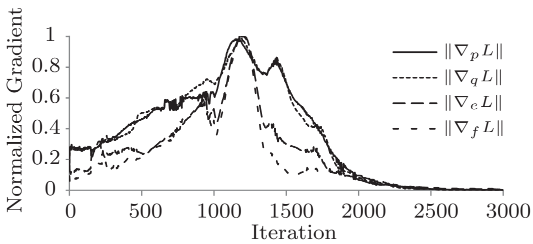

Figure 5.4: Convergence of normalized Lagrange gradients ($\nabla_x L(x, \lambda)$) to zero over iterations of algorithm 5

5.6 Conclusions

In this chapter we have presented an asynchronous, distributed algorithm for the optimization of smart grid operation. A primal dual iterative method was applied to the problem of optimally controlling DG under voltage constraints. An iterative method was presented based on successive approximation of both primal and dual variables which was shown to provide both a feasible and locally optimal solution. Both synchronous and asynchronous implementations were then presented, which were capable of achieving the same solution as the centralized method. Finally simulation results were given demonstrating the algorithm's ability to find an optimal solution.

Benefits of the proposed asynchronous, distributed smart grid control algorithm are:

- The algorithm provides a solution to smart grid control without the

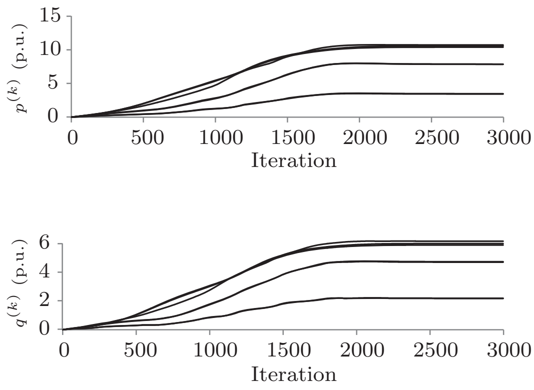

Figure 5.5: Convergence of optimal power estimates for each agent over iterations of algorithm 5

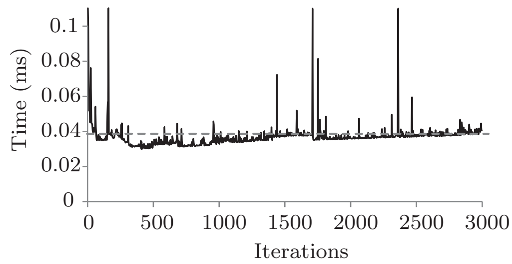

Figure 5.6: CPU time per agent per iteration over iterations of algorithm 5

need for central coordination or control,

- It provides distributed calculation of power flow, maintaining network constraints such as voltage limitations,
- The distributed approach taken is able to come to a solution through only small, frequent exchanges of data,
- Controllers are required only to communicate with neighbours with which they share a transmission line,
- Computational requirements for each agent are minimal,
- The algorithm is suitable for scenarios where local processing power is limited, but communication can be made frequently (although not necessarily reliably).

There are some aspects of the presented approach that would benefit from further research. For example, the algorithm is designed for cases where the global state and related costs are separable. Further improvements could be made to allow approximations of global information, such as through consensus. Also, the presented scenario assumes that each bus possesses an agent; an interesting addition to the algorithm would be to manage unknown elements of state where not all busses host a local controller or sensor. Finally, improvements could be made to the convergence rate of the algorithm through an improved α update process.

CHAPTER 6

Asynchronous Consensus for a Distributed Primal Dual Solution to the Smart Grid OPF Problem

The heterogeneous nature of smart grid components and the desire for smart grids to be scalable, stable and respect customer privacy have led to the need for more distributed control paradigms. In this chapter, thesis Problem 2 and its associated sub-problem of handling global values of interest are addressed by providing a distributed optimal power flow solution for a smart distribution network with separable global costs, separable non-convex constraints, and inseparable linear constraints, while considering important aspects of network operation such as distributed generation and load mismatch, and nodal voltage constraints. An asynchronous averaging consensus protocol is developed to estimate the values of inseparable global information. The consensus protocol is then combined with a fully distributed primal dual optimization utilizing an augmented Lagrange function to ensure convergence to a feasible solution with respect to power flow and power mismatch constraints. The presented algorithm uses only local and neighbourhood communication to simultaneously find the mismatch between power generation, line loss and loads, to calculate nodal voltages, and to minimize distributed costs, leading to a completely distributed solution of the global problem. An IEEE test feeder system with a reasonable number of nodes is used to illustrate the proposed method and efficiency.

6.1 Introduction

The rapid increase in smart distribution technologies such as dispatchable distributed generators (DG), storage and curtailable loads offer greater levels of controllability and observability over traditional distribution networks, which may allow for greater system stability and optimality if properly harnessed. These new opportunities come with new challenges which require new problem formulations and methods. Traditional, centralized solutions meet limitations in this regard, with new control and monitoring capabilities leading to the potential for excessive data volumes, increased computational requirements, data synchronization, latency, and privacy issues. These concerns motivate the need for improved optimization approaches, and in particular intelligent, decentralized methods which are capable of reducing centralized communication bottlenecks, distributing the processing of data, and protecting privacy, while still being capable of maintaining globally optimal or near-optimal operation.

In a report from PNNL (Pacific Northwest National Laboratory) describing a grid architecture [20] it is suggested that the existing whole grid coordination has gaps and a transition from centralized control to a hybrid central/distributed control is necessary. Furthermore, it provides insight into the need for future distribution networks to have excellent observability. Distributed approaches have been presented in the literature and often utilize features of network structure to make approximations, for example through decomposition of the network sensitivity matrix in order to form independent regions [119]. The disadvantage of utilizing completely independent regions in such approaches is that solutions are reached without considering full network state and they are prone to oscillations due to competition between controllers [2]. Hierarchical solutions, where a central coordinator takes on a leadership role, can overcome these problems by forming a multi-agent system (MAS). For example parallel optimization is performed by agents in [28, 26, 128], while global information updates such as Lagrange multipliers and aggregated load profiles are updated by a central coordinator. And in [125, 90, 92, 91] global information is aggregated by a central entity and then communicated to agents who then apply a game theoretic approach to solving the optimization. These leader-follower multi-agent systems often succeed in distributing optimization computations, but local processing is still required to perform a full optimization at each iteration, and the leader agent can still be a communication bottleneck.

It is often possible to greatly reduce the central communication burden by allowing agents to communicate with each other in a non-hierarchical manner. In such systems the leader agent becomes less relevant or even entirely redundant. Examples that utilize a leader include [87] where an auctioneer agent manages bids in the day-ahead market, and [129] where a leader agent is utilized to drive the follower agents' solutions toward a global optimum. Removing the need for a leader agent may be preferable since it removes the central point of failure. Examples of leaderless MAS are presented in [77, 32] where voltages are regulated by distributed generators acting as cooperative agents, in [78, 27] where decentralized methods of optimal reactive power control are presented, in [131] where a distributed fair load shedding algorithm is presented, and in [89] where consumer agents cooperate to perform optimal load scheduling.

A popular approach to implementing a fully distributed MAS is through the developments of a consensus protocol. Consensus protocols allow agents to reach global agreement with respect to some quantity of interest through communication only with their immediate neighbours. Consensus protocols have been extensively researched and more recently have gained attention for their applicability to distributed smart grid applications. A review of consensus protocols and their applications can be found in [160]. In [33] a continuous-time consensus protocol is developed for the regulation of voltage through droop control and reactive power sharing. In [130] frequency synchronisation of microgrids is achieved through a consesus-based algorithm. In [133] wind turbine operation in a microgrid is optimized in a distributed manner with the power imbalance in the network discovered through a consensus protocol. In [32] a consensus protocol is developed for the fair curtailment of generation in an overvoltage situation, and in [132] an average consensus algorithm is developed for load shedding in a microgrid. In [30] consensus approaches to optimal power flow (OPF), econnomic dispatch, and state estimation in the smart grid are reviewed.

A common application of consensus-based methods is the incremental cost consensus (ICC) algorithm applied to the economic dispatch problem, where local objectives are optimized while being constrained by the power mismatch within the network. In [83] distributed generator power is dispatched according to an ICC protocol. In [31] an incremental welfare consensus protocol is developed for the optimal scheduling of DG and loads. In [39] energy storage is optimally controlled through ICC. In [85] a primal-dual perturbed sub-gradient method is applied locally while averaging consensus is applied to estimate the global cost functions and constaints.

Consensus-based approaches to smart grid problems provide great improvements in terms of solving optimization problems in a purely distributed manner without the need for central communication or processing. However there are some practical problems that have not yet been well addressed in the researched applications. The first problem is the information asynchronization of the agents. Most of the developed protocols are synchronous and require some form of coordination in order to maintain the correct sequence of updates. If such a protocol is applied in an asynchronous manner the average of the consensus values within the network may drift and an average consensus cannot be reached [161]. Some research has presented asynchronous averaging algorithms such as [162] and [163], however such approaches require either pairing between agents and blocking of communication during updates, or some form of local synchronization between agents, which may lead to communication inefficiencies. The second problem is line loss, which is generally neglected entirely, or sometimes approximated as a percentage of total demand [133, 39]. These approximations will lead to inaccuracies in any arrived at solution. A final problem is the impact of power flow and voltage limits within the distribution network. Some prior works have addressed these issues including our previous paper [3] in which a distributed primal dual iterative approach is taken to solving an OPF problem. In [164] an SDP relaxation of OPF is combined with matrix decomposition to allow a distributed solution for OPF problems with linear costs. In [165] a large network is partitioned into regions and each region solves a separate OPF problem but is constrained by dummy variables at the boundary. In [82] a simplified version of the OPF problem based on linearization of constraints and convexity assumptions is solved without central coordination. In [84] a consensus-based approach is taken to the OPF problem by commissioning each agent to perform a full optimization across both its own state variables and its neighbours also. While these approaches solve, or approximately solve, the central OPF problem, they are unable to deal with inseparable components of system state such as network power mismatch. Addressing these problems is the objective of this chapter.

In this chapter we combine consensus-based approaches to handle inseparable components of the smart grid model with distributed power flow methods to produce a fully distributed (no central coordination or control), asynchronous smart grid optimization algorithm. The presented asynchronous, averaging consensus protocol is simple to implement without communication constraints such as blocking, and shares many of the benefits of synchronous consensus protocols. And the distributed, asynchronous power flow analysis allows for the discovery of line losses and voltages, ensuring the solution is feasible and optimal.

The remainder of the chapter is organized as follows: In Section 7.2 we introduce the smart grid optimization problem in terms of separable distributed generation costs, power flow and power mismatch constraints, and generation and nodal voltage limits. Section 7.4.1 presents an iterative solution based on an augmented Lagrange function, and Section 7.4 then develops a fully distributed approach based on an asynchronous consensus protocol and distributed power flow calculations. Then in Section 7.5 a simulation based case study is presented which demonstrates the operation of the asynchronous, distributed algorithm. Finally the presentation concludes with a summary of findings and a discussion of future directions in Section 7.6.

6.2 Problem Formulation

A distribution network modelled as a connected graph $(\mathcal{N}, Y)$ is considered, with buses defined by the node set $\mathcal{N}$ and edges defined by the bus admittance matrix Y. The network has a single slack bus at $0 \in \mathcal{N}$ and features controllable distributed generators (DG) at buses in $\mathcal{N}_G \subseteq \mathcal{N}$ and loads at buses in $\mathcal{N}_L \subseteq \mathcal{N}$. For convenience it is assumed that $\mathcal{N}_L = \mathcal{N} \backslash \mathcal{N}_G$ without loss of generality. For the sake of a simpler presentation of the algorithms in this chapter only a balanced network is considered, and it is assumed that each bus has all required measurements available. Application of the presented algorithms to an unbalanced distribution network is left for future study.

The goal of the smart grid optimization problem is to optimally control DG such that network operating constraints are maintained and power mismatch between generation, demand and loss is constrained to zero. The dispatchable generator operational costs are dependent on network state x, assumed continuous and convex, and are collectively given by

$$c(x) = \sum_{i \in \mathcal{N}_G} c_i(p_i, q_i), \tag{6.1}$$

where $p_i, q_i \subset x_i$ for $i \in \mathcal{N}_G$ are the active and reactive nodal powers respectively, and are constrained according to the DG capacity limitations defined by

$$p_i \in [p_i^-, p_i^+], q_i \in [q_i^-, q_i^+], \ \forall\ i \in \mathcal{N}_G. \tag{6.2}$$

Loads, which are defined by the set $\{p_i, q_i : i \in \mathcal{N}_L\}$, are measurable components of state. We denote by the vectors $p = [p_0, \ldots, p_{|\mathcal{N}|}]^\top$ and $q = [q_0, \ldots, q_{|\mathcal{N}|}]^\top$ the full set of real and reactive nodal powers. Slack bus real and reactive powers are denoted by p_0 and q_0, which ensure zero net power flow within the network.

We denote by $e = [e_0, \ldots, e_{|\mathcal{N}|}]^\top$ and $f = [f_0, \ldots, f_{|\mathcal{N}|}]^\top$ the real and imaginary components of bus voltages. Note that the real and imaginary components of voltage are used throughout the chapter rather than the more common magnitude/angle representation due to the simplification it provides to derivatives [154]. Slack bus real and imaginary voltages are denoted by e_0

and f_0 respectively, and are modelled as constant with $e_0 = 1$ and $f_0 = 0$. All other bus voltage magnitudes are constrained according to

$$(e_i^2 + f_i^2)^{\frac{1}{2}} \in [v^-, v^+], \ \forall i \in \mathcal{N}\backslash\{0\}. \tag{6.3}$$

Power flow constraints are defined in their rectangular form in terms of the real and imaginary components of voltage:

$$\begin{aligned} g(x) =& [g_{p_1}(x), \dots, g_{p_{|\mathcal{N}|}}(x), g_{q_1}(x), \dots, g_{q_{|\mathcal{N}|}}(x)]^\top, \\ g_{p_i}(x) =& \sum_{j \in \mathcal{N}} (e_i e_j G_{ij} + f_i f_j G_{ij} + f_i e_j B_{ij} - e_i f_j B_{ij}) - p_i, \\ g_{q_i}(x) =& \sum_{j \in \mathcal{N}} (f_i e_j G_{ij} - e_i f_j G_{ij} - e_i e_j B_{ij} - f_i f_j B_{ij}) - q_i, \\ \forall i \in& \mathcal{N}, \end{aligned} \tag{6.4}$$

where B and G are the real and imaginary components of the admittance matrix, Y, respectively.

The net power mismatch within the distribution network is:

$$\begin{aligned} h(x) &= [h_p(x), h_q(x)]^\top, \\ h_p(x) &= \sum_{i \in \mathcal{N}\backslash\{0\}} p_i + \tfrac{1}{2} \sum_{i \in \mathcal{N}} \sum_{j \in N_i} p_{ij}(x), \\ h_q(x) &= \sum_{i \in \mathcal{N}\backslash\{0\}} q_i + \tfrac{1}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_i} q_{ij}(x), \\ p_{ij}(x) &= G_{ij}((e_i - e_j)^2 + (f_i - f_j)^2), \\ q_{ij}(x) &= B_{ij}((e_i - e_j)^2 + (f_i - f_j)^2), \end{aligned} \tag{6.5}$$

where $p_{ij}(x)$ and $q_{ij}(x)$ are the active and reactive line losses respectively between nodes i and j. DG power is modelled as positive and loads as negative so that nodal powers sum to zero when the distribution network is operating in isolated mode (i.e. no power is imported/exported into/from the network).

The problem is formally defined as follows:

$$\begin{aligned} \text{Minimize } & c(x) \\ \text{Subject to } & g(x) = \mathbf{0}, \\ & h(x) = \mathbf{0}, \\ & x \in \mathcal{X}, \end{aligned} \tag{6.6}$$

where $\mathcal{X}$ is the set of admissible states and is defined by power generation capacity constraints (6.2) and voltage magnitude constraints (6.3). The objective of this chapter is to develop methods for calculating the power mismatch and solving (6.6) in an asynchronous, distributed manner, without a central node.

6.3 Augmented Lagrangian Optimization

Our existing work of [3] has applied augmented Lagrange optimization to solving the OPF problem. Here we summarize the important points and extend the approach to include power mismatch. The augmented Lagrange function associated with the problem of (6.6) is defined as follows:

$$\begin{aligned} L(x, \lambda, \mu) =& c(x) + \lambda^\top g(x) + \mu^\top h(x) \\ &+ \tfrac{\alpha^{(k)}}{2}\left(\|g(x)\|^2 + \|h(x)\|^2\right), x \in \mathcal{X}, \end{aligned} \tag{6.7}$$

where $\lambda = [\lambda_p, \lambda_q]^\top$ is the vector of Lagrange multipliers for the active and reactive power flow constraints, and $\mu = [\mu_p, \mu_q]^\top$ is the vector of Lagrange multipliers for the active and reactive power mismatch constraints. The penalty terms, with increasing multiplier sequence $\alpha^{(k)}$, penalize for non-zero equality constraints and assist in driving the solution towards feasibility in terms of (6.4) and (6.5).

The method of multipliers is applied to (6.7) in order to search for a solution to (6.6). Given sequences $\{\lambda^{(k)}\}$ and $\{\mu^{(k)}\}$, the iterative procedure produces the sequence $\{x^{(k)}\}$ according to

$$x^{(k+1)} = \arg\min_{x \in \mathcal{X}} L(x, \lambda^{(k)}, \mu^{(k)}). \tag{6.8}$$

We denote by $\mathcal{X}^o$ the open set containing all points of $\mathcal{X}$ excluding its boundary. Then we can say that for $x^{(k+1)} \in \mathcal{X}^o$ calculated by (6.8) the gradient satisfies

$$\|\nabla_x L(x^{(k+1)}, \lambda^{(k)}, \mu^{(k)})\| = 0. \tag{6.9}$$

To simplify the calculation of the minimum in (6.8) the preceding condition can be slackened while $x^{(k+1)} \in \mathcal{X}^o$ such that

$$\|\nabla_x L(x^{(k+1)}, \lambda^{(k)}, \mu^{(k)})\| \leq \epsilon^{(k+1)}, \tag{6.10}$$

for a sequence $\{\epsilon^{(k)}\}$ that satisfies $\epsilon^{(k)} \geq 0$ for all k and $\epsilon^{(k)} \to 0$ [155]. For points $x^{(k+1)} \notin \mathcal{X}^o$ that cannot satisfy (6.10), the minimization of (6.8) must instead be solved such that a further reduction to $L(\cdot)$ can't be made.

To achieve the minimization with respect to x, for any $x^{(k+1)} \in \mathcal{X}$, the gradient projection method [156] is taken over iterations k as follows:

$$x^{(k+1)} = P_{\mathcal{X}} \left\{ x^{(k)} - \Gamma^{(k)} \nabla_x L(x^{(k)}, \lambda^{(k)}, \mu^{(k)}) \right\}, \tag{6.11}$$

where $P_{\mathcal{X}}\{\cdot\}$ is projection on $\mathcal{X}$, and $\Gamma = \text{diag}([\gamma_i])$ is the step direction and is chosen to maximize the reduction in $L(\cdot)$. Iterations (6.11) are repeated until condition (6.10) is met, or until a reduction in $L(\cdot)$ can't be made in the case that $x^{(k+1)} \notin \mathcal{X}^o$.

Under the condition that $\{\lambda^{(k)}\}$ and $\{\mu^{(k)}\}$ are bounded, the iterations (6.8) are known to converge to a solution of (6.6) as $\alpha^{(k)} \to \infty$ [155]. The method of multipliers improves upon this result by estimating the optimal Lagrange multipliers, which we denote by λ^* and μ^*, such that under appropriate conditions, $\lambda^{(k)} \to \lambda^*$ and $\mu^{(k)} \to \mu^*$. As such we employ the projected updates

$$\lambda^{(k+1)} = P_{\Lambda} \left\{ \lambda^{(k)} + \alpha^{(k)} g(x^{(k+1)}) \right\} \tag{6.12}$$

and

$$\mu^{(k+1)} = P_{\mathcal{M}} \left\{ \mu^{(k)} + \alpha^{(k)} h(x^{(k+1)}) \right\}, \tag{6.13}$$

where $P_{\Lambda}\{\cdot\}$ is projection on $\Lambda = \{\lambda : |\lambda| \leq \lambda^+\}$ and $P_{\mathcal{M}}\{\cdot\}$ is projection on $\mathcal{M} = \{\mu : |\mu| \leq \mu^+\}$ given positive constants λ^+ and μ^+. The projection ensures that the sequences $\{\lambda^{(k)}\}$ and $\{\mu^{(k)}\}$ are bounded, which is sufficient for convergence if $\alpha^{(k)} \to \infty$ and does not impact the optimality of the solution [155]. However, if $|\lambda^*| \leq \lambda^+$, $|\mu^*| \leq \mu^+$, $x^* \in \mathcal{X}^o$, and $\nabla^2_{xx} L(x^*, \lambda^*) > 0$, then the updates (6.12) and (6.13) ensure that $\lambda^{(k)} \to \lambda^*$ and $\mu^{(k)} \to \mu^*$ when $\alpha^{(k)}$ is large enough ([155] proposition 2.4).

Appropriate values for λ^+ and μ^+ can be chosen by analysing condition (6.9) assuming $x^* \in \mathcal{X}^o$, and through simulation of a representative set of network scenarios otherwise. For the case that $x^* \in \mathcal{X}^o$ we can say that

$$\begin{aligned} &\begin{bmatrix} \lambda^* \\ \mu^* \end{bmatrix} \in \left\{ -\left(\eta(x)^\top \eta(x)\right)^{-1} \eta(x)^\top \nabla_x c(x) : x \in X^o \right\}, \\ &\eta(x) = \left[\nabla_x g(x), \nabla_x h(x)\right], \end{aligned} \tag{6.14}$$

assuming that $\eta(x)$ has rank $2|\mathcal{N}|$ [3].

6.4 Asynchronous, Distributed Solution

In the following a distributed approach to finding a solution to (6.6) is presented. An iterative algorithm is developed that applies the minimization steps of (6.11) and multiplier estimate updates of (6.12) and (6.13) with an

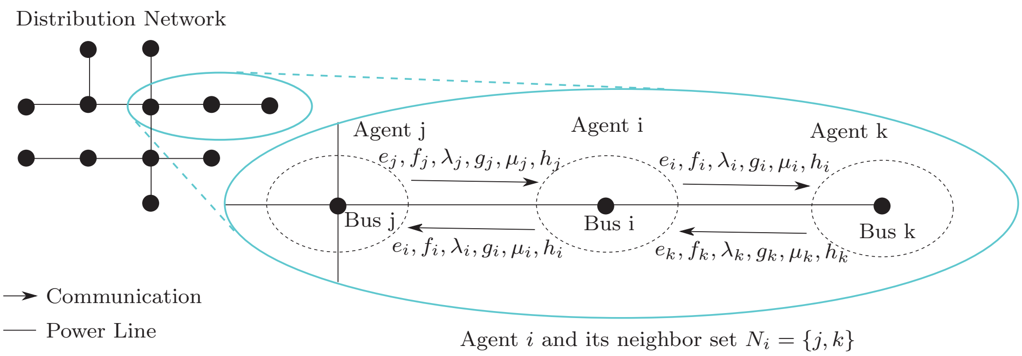

Figure 6.1: Asynchronous neighbour communication: Each agent represents a single bus within the distribution network and communicates only with its neighbours while preserving the privacy of local power production and demand.

asynchronous consensus protocol for finding the values of the coupled power mismatch $h(x)$ and its associated multiplier μ. All updates are performed asynchronously, with agents exchanging information with their immediate neighbours, an illustrations of which is presented in Figure 6.1. First an asynchronous averaging consensus protocol is presented, followed by a fully distributed implementation of (6.11). The two algorithms are then combined to provide a unified solution to the problem of (6.6).

6.4.1 Asynchronous Consensus Protocol

Consider the undirected graph $(\mathcal{N}, A)$ with node set $\mathcal{N}$, and weighted adjacency matrix $A = [a_{ij}]$. The objective of the consensus protocol is to find the average of the $|\mathcal{N}|$-dimensional vector h. Each node i holds an estimate of the average denoted by $\tilde{h}_i$, which is updated iteratively through communication only with neighbours $\mathcal{N}_i = \{j \in \mathcal{N} : a_{ij} \neq 0\}$. The basic iterative, synchronous consensus protocol can be defined as follows over iterations k:

$$\tilde{h}_i^{(k+1)} = \tilde{h}_i^{(k)} + \xi \sum_{j \in \mathcal{N}_i} (\tilde{h}_j^{(k)} - \tilde{h}_i^{(k)}), \tag{6.15}$$

where ξ is the step size. Given a step size $\xi \in (0, 1/\Delta]$ for $\Delta = \max_i(\sum_{j \neq i} a_{ij})$ and given initial estimates $\tilde{h}_i^{(0)} = h_i$, the synchronous consensus protocol will converge such that $\tilde{h}_i^{(k)} = \frac{1}{\mathcal{N}} \sum_j h_j$, $\forall i \in \mathcal{N}$ as $k \to \infty$ [160].

Averaging consensus relies on maintaining the average value across the network after each iteration k. Specifically

$$\sum_{i \in \mathcal{N}} \tilde{h}_i^{(k)} = \sum_{i \in \mathcal{N}} h_i, \ \forall k. \tag{6.16}$$

If (6.15) is performed asynchronously, for example if not all nodes are updated at each iteration, then the consensus result will not give the average of the initial condition and (6.16) will not be maintained [161]. Asynchronous, averaging consensus protocols have been presented in the literature and typically employ a Symmetric Gossip strategy (a description of which is presented in [166]). However existing implementations of this strategy require some form of local synchronization [163], explicit pairing (agreement between neighbours as to who controls communication) [167] or blocking [162]. Next we present a simple asynchronous averaging consensus protocol that avoids the need for these mechanisms and possesses the following properties:

- *Distributed*: No central controller or leader agent.
- *Asynchronous*: No inter-agent synchronization.
- *Implicit pairing*: No blocking or explicit pairing.
- *Averaging*: Average consensus is reached.
- *Tracking*: Average is tracked as network state changes.

If protocol (6.15) is applied for a single update, for example when a single node i performs an update at iteration k, then after the update is complete the sum of value estimates in (6.16) will be incorrect by a factor of $\xi \sum_{j \in \mathcal{N}_i} (\tilde{h}_j^{(k)} - \tilde{h}_i^{(k)})$. To maintain the average an amount can be subtracted from each neighbour's estimate. This amount does not have to be subtracted immediately and can be queued by each neighbour and subtracted when it next performs an update. We specify the consensus correction variable w_i to store this value and the resulting protocol is presented in algorithm 9.

Algorithm 9 maintains the average

$$\sum_{i \in \mathcal{N}} (\tilde{h}_i^{(k)} + w_i^{(k)}) = \sum_{i \in \mathcal{N}} h_i, \ \forall k, \tag{6.19}$$

even though the asynchronous nature of the algorithm, defined by the update set $\mathcal{S}^{(k)}$, implies some nodes may not be updated at iteration k.

In practice, we can't assume that updates for w_i will not occur in parallel. Therefore all such requests should be queued. Additionally, since a single failure to update w_i will cause the average to be shifted and condition (6.19) to be breached, it is important that these updates require an acknowledge message from each neighbour.

Given non-empty update set $S^{(k)}$, and ignoring constraint (6.19) such that $w_i^{(k)} = 0, \ \forall i, k$, the update (7.20) can be synchronously defined in matrix

Algorithm 6 Asynchronous Consensus Protocol

Initialize $\tilde{h}_i^{(0)} = h_i, w_i^{(0)} = 0,\ \forall i \in \mathcal{N}$.
For $k = 1, 2, \ldots$**:**

1. Choose the set of nodes $\mathcal{S}^{(k)} \subseteq \mathcal{N}$ to update.
2. **For each** $i \in \mathcal{S}^{(k)}$**:**

   2.1 Update average estimate:

   $$\tilde{h}_i^{(k+1)} = \tilde{h}_i^{(k)} + \xi \sum_{j \in \mathcal{N}_i} (\tilde{h}_j^{(k)} - \tilde{h}_i^{(k)}) - w_i^{(k)}. \tag{6.17}$$

   2.2 **For each** $j \in \mathcal{N}_i$**:**

   $$w_j^{(k+1)} = w_j^{(k)} + \xi(\tilde{h}_j^{(k)} - \tilde{h}_i^{(k)}). \tag{6.18}$$

   2.3 Reset $w_i^{(k+1)} = 0$.

---

form as follows:

$$\tilde{h}^{(k)} = \left(\prod_{l=0}^{k} P^{(l)} \right) h, \tag{6.20}$$

where P is the Perron matrix and is defined as

$$\begin{aligned} P^{(k)} &= I - \xi D^{(k)}, \\ D_{ij}^{(k)} &= \begin{cases} |\mathcal{N}_i| & j = i \text{ and } i \in \mathcal{S}^{(k)}, \\ -1 & j \in \mathcal{N}_i \text{ and } i \in \mathcal{S}^{(k)}, \\ 0 & \text{otherwise}, \end{cases} \end{aligned} \tag{6.21}$$

where I is the identity matrix, and $|\mathcal{N}_i|$ is the cardinality of the neighbour set of node i.

Theorem 6.4.1. Assume there exists a positive constant m such that for the sequence $\{k, k+1, \ldots, k+m\}$ the graph associated with the matrix $\prod_{l=k}^{k+m} P^{(l)}$ is fully connected for all k. Then under the iterations specified by algorithm 9, and given step size $0 < \xi < 1/\Delta$ for $\Delta = \max_i |\mathcal{N}_i|$, all local estimates $\tilde{h}_i$ converge to the average of the initial values h_i as $k \to \infty$, such that

$$\tilde{h}_i^{(k)} \to \tfrac{1}{|\mathcal{N}|} \sum_{i \in \mathcal{N}} h_i. \tag{6.22}$$

Remark. The assumption on the connectivity of the product of Peron matrices can be understood intuitively as there being a uniform upper bound on the number of iterations required before information from one agent can propagate through the network to any other agent.

The following proof is adapted for the asynchronous case from the synchronous convergence analysis provided by [160].

Proof. It can be seen from the updates in (6.20) that a consensus is reached if the limit $\lim_{k\to\infty}\prod_{l=k}^{k+m} P^{(l)}$ exists. Having $0 < \xi < 1/\Delta$ gives $D_{ii} < 1$, and $0 < P_{ii} < 1$, $\forall i$, and also $0 < P_{ij}, i \neq j$. It then follows that the matrix $\prod_{l=k}^{k+m} P^{(l)}$ has the follows properties:

1. All diagonal elements are non-negative,
2. All off-diagonal elements are positive,
3. The digraph associated with the matrix is strongly connected.

Therefore the matrix is primitive and according to Lemma 4 from [160] we can say that $\lim_{k\to\infty}\prod_{l=k}^{k+m} P^{(l)}$ exists. It then follows that $\tilde{h}_i^{(k)} = \tilde{h}_j^{(k)}$, $\forall i, j \in \mathcal{N}$ as $k \to \infty$.

The average conservation variable w_i can be considered a bias in the updates of (7.20), which does not affect the stability analysis of the algorithm [160]. Given the consensus of the variables $\tilde{h}_i^{(k)}$ and since each node is updated such that $w_i^{(k)} = 0$ at least every m iterations, it follows from (7.21) that $w_i^{(k)} \to 0$ as $k \to \infty$. Then (6.19) gives $\sum_{i\in\mathcal{N}}\tilde{h}_i^{(k)} = \sum_{i\in\mathcal{N}} h_i$ and from consensus $\tilde{h}_i^{(k)} \to \frac{1}{N}\sum_{i\in\mathcal{N}} h_i$, $\forall i \in \mathcal{N}$.

□

The preceding proof states that a bias does not affect the stability analysis. This can be seen by following the progression of the average condition with biases added:

$$\sum_{i\in\mathcal{N}}\tilde{h}_i^{(k)} = \sum_{i\in\mathcal{N}}\left(h_i + \sum_{j=0}^{k-1} b_i^{(j)}\right). \tag{6.23}$$

Therefore adding a bias is equivalent to modifying the initial state in terms of (6.16). It follows that the average condition will converge if $\sum_{i\in\mathcal{N}}\sum_{j=0}^{k-1} b_i^{(j)} \to C$, for some $C \in (-\infty, \infty)$.

For an analysis of the optimal choice of tuning parameter ξ the reader is referred to [168] where an eigenvalue analysis of the Laplacian provides both admissible and optimal values. This applies to the synchronous case, but extends to the asynchronous case by replacing the Laplacian with its expectation.

6.4.2 Asynchronous, Distributed Algorithm

The update of (6.13) must be made centrally since global information is required for the calculation which can't be separated. To convert the algorithm to a distributed form the inseparable power mismatch constraint $h(x)$ is calculated through the asynchronous consensus protocol of algorithm 9. The local estimate of power mismatch is $\tilde{h}_i^{(k)} = [\tilde{h}_{p_i}^{(k)}, \tilde{h}_{q_i}^{(k)}]^\top$ for node i at iteration k. The power mismatch estimate update is then given by

$$\begin{aligned}
\tilde{h}_i^{(k+1)} =& \tilde{h}_i^{(k)} + \xi_h \sum_{j\in\mathcal{N}_i} (\tilde{h}_j^{(k)} - \tilde{h}_i^{(k)}) - w_i \\
& + s_i^{(k+1)} - s_i^{(k)} + \sum_{j\in\mathcal{N}_i} (s_{ij}(x^{(k+1)}) - s_{ij}(x^{(k)})), \\
w_j^{(k+1)} =& w_j^{(k)} + \xi_h (\tilde{h}_j^{(k)} - \tilde{h}_i^{(k)}), \ \forall j \in \mathcal{N}_i, \\
w_i^{(k+1)} =& 0,
\end{aligned} \tag{6.24}$$

where nodal complex power is defined as $s_i = [p_i, q_i]^\top$ and line loss as $s_{ij}(x) = [p_{ij}(x), q_{ij}(x)]^\top$.

A similar approach is taken for the approximation of the power mismatch multiplier μ, however since average consensus is not required a standard asynchronous consensus protocol is used without the average maintaining terms w_i:

$$\tilde{\mu}_i^{(k+1)} = \tilde{\mu}_i^{(k)} + \xi_\mu \sum_{j\in\mathcal{N}_i} (\tilde{\mu}_j^{(k)} - \tilde{\mu}_i^{(k)}) + \alpha \tilde{h}_i^{(k+1)}. \tag{6.25}$$

The bias terms present in (6.24) and (6.25), are $(s_i^{(k+1)} - s_i^{(k)}) + \sum_{j\in\mathcal{N}_i} (s_{ij}^{(k+1)} - s_{ij}^{(k)})$ and $\alpha \tilde{h}_i^{(k+1)}$ respectively. These terms shift the global average according to the change in state at iteration k, but they do not impact the stability of algorithm 9 and therefore they ensure that the estimates $\tilde{h}$ and $\tilde{\mu}$ track the global values. Therefore, so long as the bias terms' sums converge, the estimates will converge to the true global values, that is $\tilde{h}_i^{(k)} \to \frac{1}{N} h(x^{(k)})$ and $\tilde{\mu}_i^{(k)} \to \mu$ for all $i \in \mathcal{N}$, where μ is the global estimate of the optimal multiplier μ^*. In this way an approximation can be made for the gradient descent steps of (6.11):

$$\begin{aligned}
x_i^{(k+1)} = P_{\mathcal{X}} \Big\{ x_i^{(k)} - \gamma_i^{(k)} \Big(& \nabla_{x_i} c_i(x_i^{(k)}) \\
& + \nabla_{x_i} h(x^{(k)}) (\tilde{\mu}_i^{(k)} + \alpha \tilde{h}_i^{(k)}) \\
& + \nabla_{x_i} g(x^{(k)}) (\lambda^{(k)} + \alpha g(x^{(k)})) \Big) \Big\},
\end{aligned} \tag{6.26}$$

where $\nabla_{x_i} h(x^{(k)})$ and $\nabla_{x_i} g(x^{(k)})$ are calculable locally and are non-zero only for elements corresponding to node i and its neighbours [3]. This modification makes a fully distributed solution possible. The asynchronous, distributed algorithm for finding a solution to (6.6) is presented in algorithm 7, where $h_i(x) = s_i + \frac{1}{2} \sum_{j \in \mathcal{N}_i} s_{ij}(x)$.

Algorithm 7 Asynchronous Distributed Algorithm

Initialize

Power mismatch: $\tilde{h}_i^{(0)} = h_i(x_i^{(0)})$, $\tilde{\mu}_i^{(0)} = 0, \forall i \in \mathcal{N}$.

Generator power: $[p_i, q_i]^\top = \arg\min\{c_i(x_i^{(0)})\}, \forall i \in \mathcal{N}_G$.

Voltages: $e_i^{(0)} = 1$, $f_i^{(0)} = 0, \forall i \in \mathcal{N}$.

For $k = 1, 2, \ldots$**:**

1. Choose the set of nodes $\mathcal{S}^{(k)} \subseteq \mathcal{N}$ to update.
2. **For each** $i \in \mathcal{S}^{(k)}$**:**

   2.1 Update primal variable $x_i^{(k)}$ (6.26).

   2.2 Update multiplier $\lambda_i^{(k)}$ (6.12).

   2.3 Update power mismatch estimate $\tilde{h}_i^{(k)}$ (6.24).

   2.4 Update multiplier estimate $\tilde{\mu}_i^{(k)}$ (6.25).

   2.5 Increase penalty multiplier: $\alpha_i^{(k+1)} = \beta \alpha_i^{(k)}$.

6.5 Simulation Results

We consider the case of a distribution network featuring numerous distributed generators (DG) capable of operating in an isolated mode; that is, operating such that power generated within the network is able to match all loads and losses within the network. Furthermore, each bus in the network is equipped with an agent that has knowledge of nodal DG or load power, and has a communication interface with neighbouring agents. The test network is a balanced three phase implementation of a 35 bus subnetwork of the IEEE 123 node test feeder system [152]. Only buses 1 to 34 were included along with bus 149 configured as a slack bus. Dispatchable generators, each with a 500kW capacity, were attached to buses 1, 3, 8, 14, 18, 26 and 29, and their associated agents were equipped with cost functions $c_i(p_i, q_i) = 0.5p_i^2 + 0.5q_i^2$. The

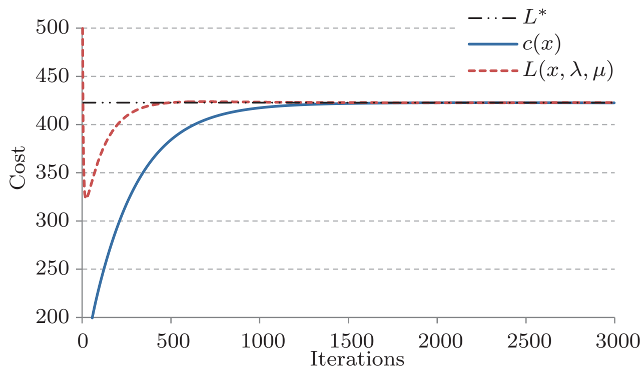

Figure 6.2: Convergence of Lagrange function and cost function to solution of centralized OPF, L^*, over iterations of algorithm 7.

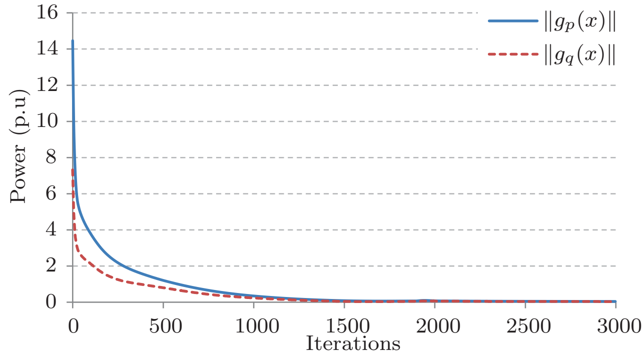

Figure 6.3: Convergence of power flow constraints, $g(x)$, to zero over iterations of algorithm 7.

base voltage was set to 1p.u.=120V, voltage magnitudes were constrained according to (6.3) with $[v^-, v^+] = [0.95, 1.05]$p.u, and base power was set to 1p.u.=10kVA.

Algorithm 7 was applied to the test network, initialized with a random feasible state with cost of 512.5, and resulted in the convergence of the Lagrange and cost functions to the cost of a centralized OPF solution as presented in Figure 6.2. This convergence also implies that $g(x^{(k)}) \to 0$ and $h(x^{(k)}) \to 0$ which is further evident in Figure 6.3 which shows the norm of the power flow constraints converging to zero.

Figure 6.4 shows the estimate of the power mismatch calculated through consensus by each agent (solid black lines), and the average of the true power mismatch (broken red line). It can be seen that the asynchronous consensus

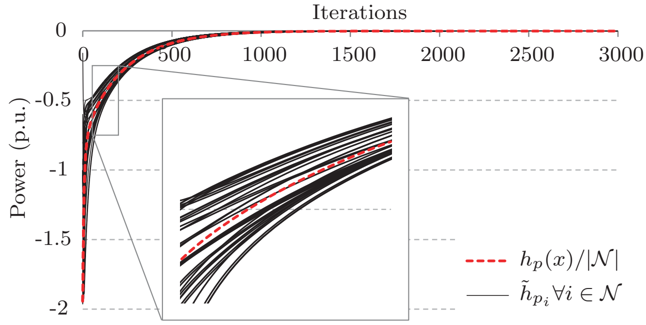

Figure 6.4: Consensus tracking of average active power mismatch by agents through algorithm 9, and convergence of power mismatch to zero over iterations of algorithm 7.

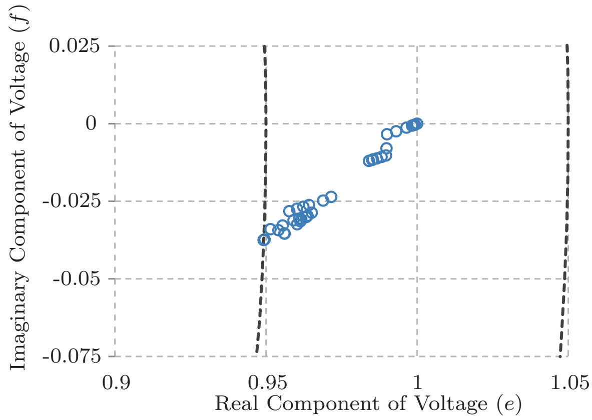

Figure 6.5: Nodal voltages after convergence of algorithm 7.

protocol of 9 successfully converges on and tracks the true power mismatch $h_p(x)$ without drifting (similar results were found for $h_q(x)$). Furthermore, algorithm 7 successfully reduces the power mismatch, including line loss, to zero. As such all equality constraints are satisfied.

Figure 6.5 presents the final voltages of the network which are clearly within voltage magnitude limits. The voltage at bus 24 has been constrained according to (6.3) (refer to the marker on the 0.95p.u. lower bound) indicating that the solution lies on the boundary of $\mathcal{X}$. Figure 7.6 presents the estimates of optimal generated power as iterations of algorithm 7 progress, and are within DG capacities according to (6.2). Therefore, the arrived at solution is feasible; that is, $x \in \mathcal{X}$, $g(x) = 0$, $h(x) = 0$.

Finally, Figure 6.7 presents the normalized gradients of the Lagrange

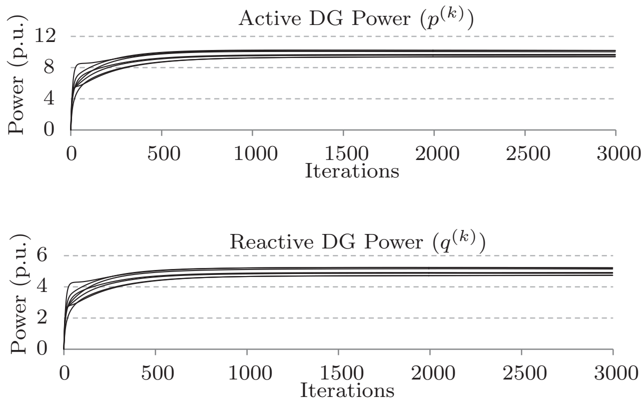

Figure 6.6: Convergence of optimal DG power estimates for each agent over iterations of algorithm 7.

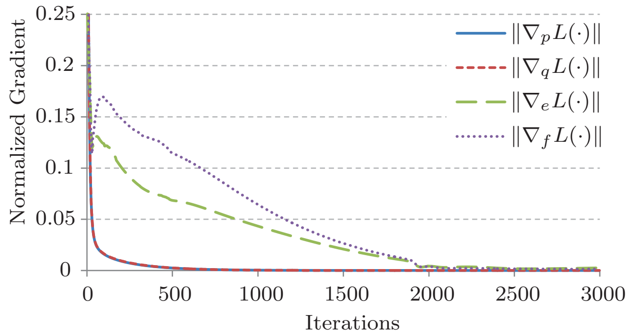

Figure 6.7: Convergence of normalized Lagrange gradients to zero over iterations of algorithm 7.

function. As described above, the voltage at bus 24 lies on the boundary of $\mathcal{X}$ and due to the projection operation of (6.11) the gradient with respect to e_{24} and f_{24} cannot satisfy (6.10) as $\epsilon^{(k)} \to 0$. As such, these elements of the gradient have been neglected in the presentation of Figure 6.7. Clearly all other gradients successfully approach zero in accordance with (6.10) such that a local optimum is reached. It should be noted that in many tested scenarios, including the one presented here, the sequence $\{\epsilon^{(k)}\}$ could be chosen such that only a single gradient descent step was required at each iteration.

6.6 Conclusion

We have presented an asynchronous, distributed algorithm for optimal DG control in the Smart Grid, which enforces zero power mismatch between generation, load and line loss, and enforces nodal voltage constraints. An asynchronous averaging consensus protocol was developed and applied to the discovery of power mismatch within the network which is otherwise not calculable by agents since they only possess local and neighbourhood information. The protocol was able to maintain a system average without any synchronization between agents, and was able to drive each agent to converge on the average power mismatch value. The asynchronous, distributed optimization algorithm was combined with the asynchronous consensus protocol to deliver a feasible and locally optimal solution requiring no central coordination or control. Finally, the application of the combined asynchronous, distributed algorithms was presented through a case study, demonstrating its ability to converge to a feasible solution in terms of power flow, power mismatch, and nodal voltage constraints, and to reach a locally optimal solution.

Chapter 7 has been removed for copyright or proprietary reasons.

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CHAPTER 8

Conclusion

8.1 Summary and Discussion

This thesis considered the case of modern and future power distribution systems where a number of challenges are present due to the high penetration of distributed generators (DG) leading to reverse power flow, and the increased observability and controllability of nodal agents leading to high data volumes. The presented problems pose a significant concern to system security, reliability and stability if not properly addressed. However the presented research takes advantage of the advanced nodal monitoring and control offered by smart grid components including DG, storage and controllable demands to overcome these issues and improve system performance. Innovative distributed methods were given to maintain network operational constraints, and improve optimal power flow, while allowing for the realistic limitation of reduced network information available to individual smart grid agents.

Two network partitioning strategies were considered to address two important issues in the smart grid agent communication and control system: Optimal DG utilisation, and network state estimation error reduction. The first strategy demonstrated the importance of the balance of smart grid components within a controller's area of influence. The presented power balanced partitioning algorithm was shown to improve the DG management structure such that DG utilisation was maximised, high voltage grid imports were minimised, and distribution feeder voltage profiles were improved. The second strategy was developed in order to support an optimal power flow (OPF) approach based on sub-network state approximation. The power flow based partitioning strategy was successful in reducing estimation errors allowing the OPF solution to reach a near optimal point.

Two distributed, asynchronous optimal power flow (OPF) strategies were presented that utilised the partitioning algorithms and an agent per bus system respectively. The first OPF strategy operated on a two stage system composed of algorithms for a central coordinator and distributed agents. Each agent performed a time-varying OPF optimisation through the application of approximate dynamic programming (ADP). The APD algorithm was shown to provide a near optimal solution to the problem of storage scheduling, and scaled well as the buses, DG, storage and loads increased. The central coordinator was demonstrated to refine state estimation and reduce agent error, while alleviating distributed agents from the burden of high dimensional state.

The second presented distributed, asynchronous OPF strategy removed the need for the central coordinator and addressed the global problem through a fully decentralised system of agents. The strategy was shown to possess low computational requirements and therefore can be easily implemented by small scale distributed controllers. Furthermore, the communication structure was able to reduce communication to small neighbourhoods, therefore reducing the overall data transmission requirements of the smart grid communication system. This fully distributed approach was proven and demonstrated to reach a local optimum for the global problem in each of three distinct cases: A high-voltage grid connected distribution network, an islanded distribution network, and a smart building low-voltage DC network. To support the algorithms of the islanded and smart-building case, a new asynchronous, consensus protocol was presented that was proven to maintain a system average without requiring any synchronisation mechanisms between agents, and was successful in driving each agent to the correct average value. To support the implementation of the asynchronous, distributed OPF strategy an asynchronous data exchange protocol was presented that was able to initiate, conduct and conclude optimisations sessions.

The following sections summarise the contributions of this thesis described above, and suggest future directions for research that builds upon the approaches developed.

8.2 Contributions

The primary contributes of this thesis can be divided into three components: The partitioning strategies applied to improve network manageability, the distributed optimisation algorithms developed for smart grid scheduling and control, and the protocols applied to smart grid multi-agent networks in order to enable the optimisation algorithms. The following sections elaborate on these components providing further detail for the respective contributions.

8.2.1 Partitioning Strategies

The first problem addressed was the logical and communication structure of the network utilised for distributed optimisation. In all cases studied within this thesis, buses within the network were assigned to an agent to form a multi-agent system (MAS) to manage the flow of information utilised by the distributed optimisation algorithms. The MAS structure provided the benefit of regionalising the smart grid such that processing can be performed locally based on local knowledge, and therefore communication and data processing burdens can be reduced, and system adaptability and robustness can be improved.

Three new strategies were presented for the partitioning of each network into logical groups:

Power balanced partitioning into zones: In order to better utilise distributed sources of power a novel partitioning strategy was developed that divided a distribution network into dynamic zones. Divisions were made according to the impact of a change in voltage at one bus on the voltage of another. This impact was combined with the balance of power generated and consumed within the partition (refer to Chapter 3). This approach was demonstrated to have the benefit of each agent managing DG that is close to the loads it is supplying and was able to reduce the power imported into the zone resulting in more efficient use of available DG and improved voltage profiles.

Power flow based ϵ decomposition: To address the problem of state estimation error typically present in a partitioned network, a strategy was developed that analysed errors introduced through changing network state. The network was partitioned so as to minimise voltage calculation error over a time window through the linearisation of power flow calculations and the use of power generation and load forecasts (refer to Chapter 4). This strategy was shown to allow optimisation methods based on network state approximation to be performed with reduced error.

Fully distributed: To maximise the autonomy of agents while minimising communication and data processing burdens, an agent per bus approach was considered. Each bus within the network was assigned its own agent to form a MAS communication network that exactly overlaid the power distribution network; in this fully distributed case the central agent was removed and all calculations depended entirely on communication with direct neighbours (refer to Chapters 5, 6 and 7). This approach provided the greatest flexibility since network structure changes could be accommodated without the need for a central entity to redefine the logical and communication network structure.

8.2.2 Optimisation Algorithms

The second problem addressed was the OPF within distribution networks based on the distributed observation and control of nodal values. Both approximate and exact iterative solutions were presented, and both centrally coordinated and fully distributed MAS were utilised. Each solution considered the optimisation of the whole distribution network and relied on local network state observations and control at the agent level.

Coordinated, distributed OPF with state estimation: In order to achieve a global near-optimal solution of a large network with high-dimensional, time varying state, a novel iterative state estimation and optimisation approach was developed. The approach provided approximate, distributed power flow calculations which were applied to Approximate Dynamic Programmes (ADP) of subsets of the global problem (refer to Chapter 4). Given the potential for large data volumes due to the high dimensional state of a network with numerous agents impacted by decisions over time, the OPF problem begins to meet limitations in terms of the timeliness and asynchronous availability of data. Through centrally coordinated refinements the local ADP solutions were shown to improve optimal scheduling of DG output and storage while allowing agents to act only on their neighbourhood.

Fully distributed primal dual iterations: For the sake of a more flexible solution in terms of adaptability and robustness, a fully decentalised method was developed that was able to reach an optimum of the global problem. The distribution network OPF problem was separated into a set of localised calculations leading to a distributed and asynchronous iterative algorithm. The algorithm was adapted to three separate scenarios, each of which addressed different aspects of its practical application:

1. The basic OPF problem of controllable DG and fixed loads in a distribution network were considered (refer to Chapter 5). Since power flow calculations are typically performed through calculations based on full network state, distributed optimisation solutions presented in the literature are often solved by approximating line loss. In contrast, the asynchronous, distributed OPF algorithm presented in this thesis, was able to accurately take into account line loss within the network

through the application of primal dual iterations based on only local and neighbourhood values.

2. The case of an islanded distribution network was considered such that power generated within the network was made to match the power consumed (refer to 6). The calculation of the power mismatch between DG, line loss and loads requires global knowledge, providing a challenge for the fully distributed and asynchronous solution. Through the development of a new asynchronous averaging consensus algorithm the iterative solution was able to estimate the power mismatch and ultimately come to a solution of the islanded distribution network OPF problem.

3. The optimal operation of a DC home energy management system (HEMS) and the practical requirements for asynchronous information exchange needed by the distributed OPF algorithm were considered (refer to Chapter 7). Given the asynchronous nature of the distributed algorithm consideration must be given to the coordination of agents such that optimisation can begin at appropriate times and a mutual conclusion to the algorithm can be agreed upon. Through development of the Home Energy Management Multi-Agent (HEMMA) protocol agents within the HEMS were able to coordinate optimisation session initialisation, execution and conclusion without the need for a central leader agent.

8.2.3 Protocols

The third problem addressed within this thesis was need for coordination and communication protocols that will allow the practical implementation of the presented algorithms. In particular, given that the algorithms are typically asynchronous, there is a need to consider the means of coordination without synchronisation, and error handling. The following two algorithms allow for asynchronous exchange of data and the reaching of global agreement between distributed agents.

Asynchronous Averaging Consensus Protocol To allow for the practical discovery of global values within a network, an asynchronous averaging consensus protocol was developed. In contrast with other asynchronous consensus protocols presented in the literature the approach presented in this thesis relies on implicit pairing and does not require any blocking of protocol messages or local synchronisation while performing an asynchronous message exchange. The protocol was proven to converge with very similar properties to its synchronous averaging consensus protocol counterpart.

Home Energy Management Multi-Agent (HEMMA) Protocol The HEMMA protocol was developed to support the implementation of the presented asynchronous, distributed OPF solutions. The protocol was demonstrated to facilitate network discovery, agent data exchange, and optimisation session management.

8.3 Future Directions

Due to time constraints and other limitations, there are many interesting issues that are not included in the scope of my PhD study. Some of the methods presented could therefore benefit from further research, refinement and extension. The following suggestions are made for future directions:

1. Each algorithm presented has been built on the steady state analysis of the network and therefore has an implicit assumption that the network will have time to reach steady state prior to reapplication of the optimisation. As such, it may be beneficial to study the impact of control decisions on the transient behaviour of the network.

2. The logical structure in each presented MAS assumes that an agent is available for measurement or control at each bus. An interesting addition to the research would be the inclusion of unobservable elements within the network.

3. In the case of distributed OPF based on the method of multipliers, numerous parameters must be chosen. Algorithm performance may be improved through a deeper analysis of optimal parameter selection, which has not been included in this thesis.

4. The OPF algorithms presented in Chapters 5, 6 and 7 consider optimisation problems where the impact of present decisions do not impact future state. It would be practical to extend these algorithms to problems including timely elements such as storage and curtailable loads, for example through the addition of ADP.

5. The OPF algorithms presented in Chapters 5, 6 and 7 consider constant power and constant current loads. It would improve the applicability of the algorithms to consider stochastic loads and assess the associated probability distributions.

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